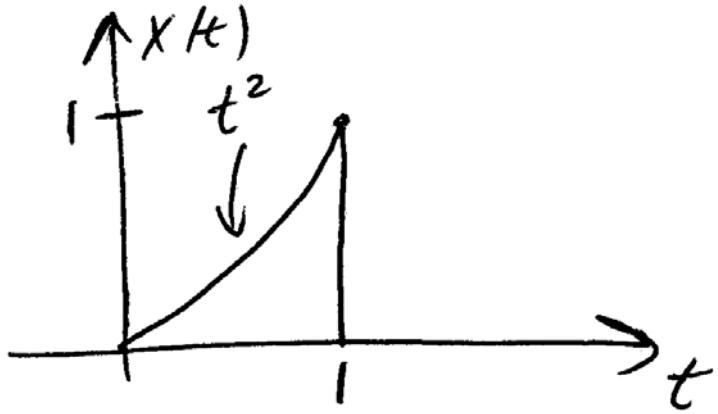
Example- Find the Fourier transform of the signal shown.



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{1} t^2 e^{-j\omega t}\, dt$$

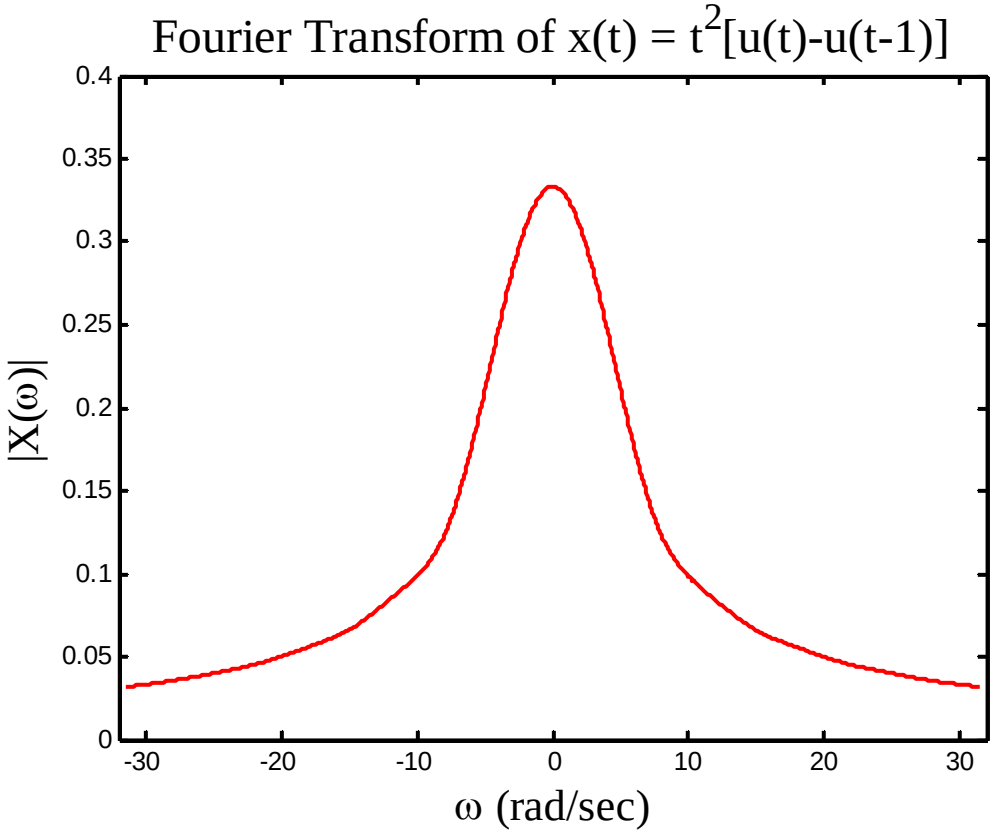
$$= \frac{e^{-j\omega t}}{-j\omega}\left(t^2 - \frac{2t}{-j\omega} + \frac{2}{(-j\omega)^2}\right)\Bigg|_{t=0}^{1}$$

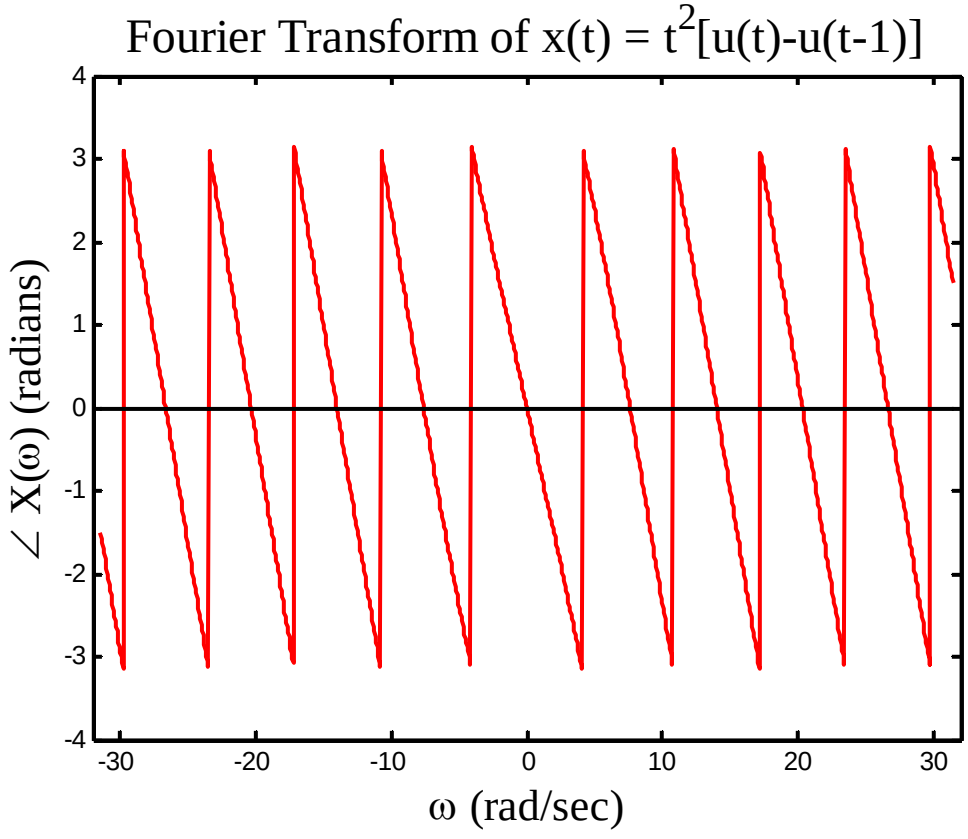
$$= \left(\frac{e^{-j\omega}}{-j\omega}\left(1 - \frac{2}{-j\omega} + \frac{2}{(-j\omega)^2}\right)\right)$$

$$- \left(\frac{1}{-j\omega}\left(0 - 0 + \frac{2}{(-j\omega)^2}\right)\right)$$

$$X(\omega) = \frac{e^{-j\omega}}{-j\omega}\left(1 + \frac{2}{j\omega} - \frac{2}{\omega^2}\right) - \frac{2}{j\omega^3}$$

```
% Chapter 3 Fourier Transform of t^2 pulse
% (chap3_fourier_tran_tsqr.m)
%
close all;clear;clc;
% initialize frequency and FT vectors
w = zeros(1,1001);
Xw = zeros(1,1001);
zw = zeros(1,1001); % zero axis
% Calculate frequencies and Fourier Transform
for k=1:1001,
    w(k) = (k-501)*pi/50;
    if(w(k) == 0),
        Xw(k) = 1/3; % avoid divide by zero
    else
        Xw(k) = exp(-1j*w(k))/(-1j*w(k))*(1+2/1j/w(k)-2/w(k)/w(k))...
        -2/(1j*w(k)^3);
    end
end
% Plot magnitude of Fourier transform
plot(w,abs(Xw),'r'),axis([-32 32 0 0.4]),
title('Fourier Transform of x(t) = t^2[u(t)-u(t-1)]','fontsize',...
    18,'fontname','times'),
ylabel('|X(\omega)|','fontsize',16,'fontname','times');
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times'),
% Plot angle of Fourier transform
figure
plot(w,angle(Xw),'r',w,zw,'k'),axis([-32 32 -4 4]),
title('Fourier Transform of x(t) = t^2[u(t)-u(t-1)]','fontsize',...
    18,'fontname','times'),
ylabel('\angle X(\omega) (radians)','fontsize',16,'fontname','times');
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',18)
set(findobj('type','axes'),'linewidth',2)
```


Fourier Transform of x(t) = t2[u(t)-u(t-1)]
|X(ω)|
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-30
-20
-10
0
10
20
30
ω (rad/sec)


Fourier Transform of x(t) = t2[u(t)-u(t-1)]
∠ X(ω) (radians)
4
3
2
1
0
-1
-2
-3
-4
-30
-20
-10
0
10
20
30
ω (rad/sec)