Example- Find the Fourier transform of the rectangular pulse shown.
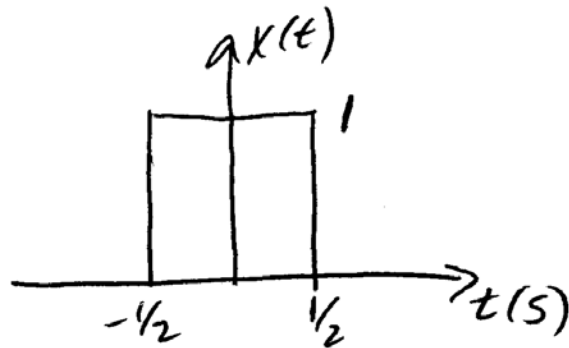
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-1/2}^{1/2} e^{-j\omega t}\, dt$$

$$= \left. \frac{e^{-j\omega t}}{-j\omega}\right|_{-1/2}^{1/2} = \frac{e^{-j\omega/2} - e^{+j\omega/2}}{-j\omega}$$

$$= \frac{\cos(\omega/2) - j\sin(\omega/2) - (\cos(\omega/2) + j\sin(\omega/2))}{-j\omega}$$

$$= \frac{-j\,2\sin(\omega/2)}{-j\omega} = \frac{\sin(\omega/2)}{(\omega/2)}$$
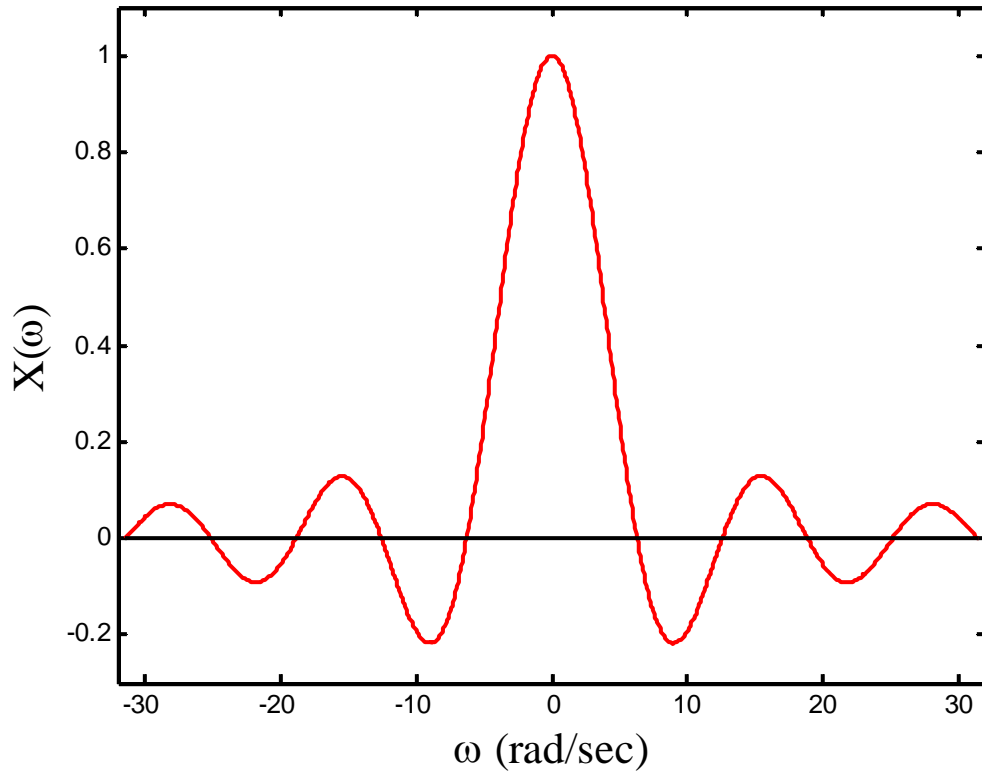
$$X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\text{where } \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \qquad \frac{\omega}{2} = \pi x$$
$$x = \frac{\omega}{2\pi}$$

```matlab
% Chapter 3 Fourier Transform of rectangular pulse
% (chap3_fourier_tran_rect_pulse.m)
%
close all;clear;clc;
% initialize frequency and FT vectors
w = zeros(1,1001);
Xw = zeros(1,1001);
zw = zeros(1,1001); % zero axis
% Calculate frequencies and Fourier Transform
for k=1:1001,
    w(k) = (k-501)*pi/50;
    if(w(k) == 0),
        Xw(k) = 1; % avoid divide by zero
    else
        Xw(k) = sin(w(k)/2)/(w(k)/2);
    end
end
for k2=1:1001,
    if(w(k2) < 0),
        Xang(k2) = -angle(Xw(k2));
    else
        Xang(k2) = angle(Xw(k2));
    end
end
% Plot Fourier transform (unusual that it is entirely real)
plot(w,Xw,'r',w,zw,'k'),axis([-32 32 -0.3 1.1]),
title('Fourier Transform of Rectangular Pulse','fontsize',...
    18,'fontname','times'),
ylabel('X(\omega)','fontsize',16,'fontname','times'),
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times'),
% Plot magnitude of Fourier transform
figure
plot(w,abs(Xw),'r'),axis([-32 32 0 1.1]),
title('Fourier Transform of Rectangular Pulse','fontsize',...
    18,'fontname','times'),
ylabel('|X(\omega)|','fontsize',16,'fontname','times');
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times'),
% Plot angle of Fourier transform
figure
plot(w,Xang,'r',w,zw,'k'),axis([-32 32 -4 4]),
title('Fourier Transform of Rectangular Pulse','fontsize',...
    18,'fontname','times'),
ylabel('\angle X(\omega) (radians)','fontsize',16,'fontname','times');
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',18)
set(findobj('type','axes'),'linewidth',2)
```

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