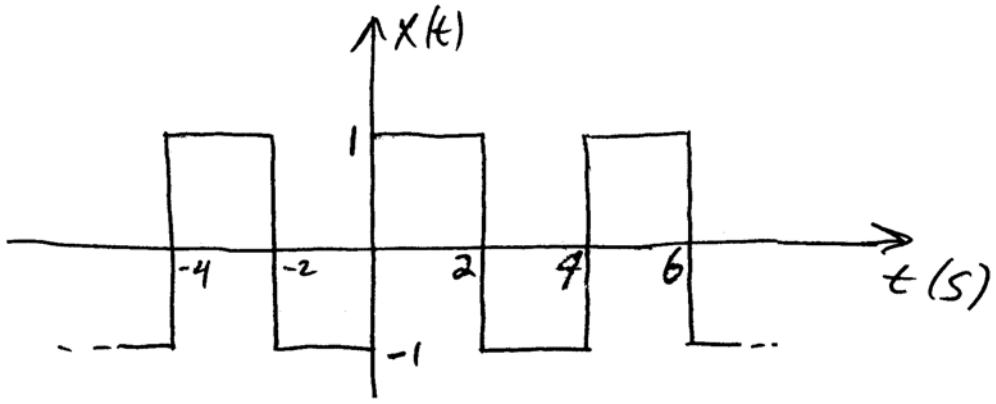
Ex. Calculate the complex exponential Fourier series for the binary sine wave shown.



→ odd function, $T = 4s$, $\omega_0 = \frac{2\pi}{T} = \pi/2 \ \frac{rad}{s}$

$$C_0 = \frac{1}{4}\int_0^4 x(t)\,dt = 1/4\int_0^2 1\,dt + 1/4\int_2^4 (-1)\,dt = \underline{\underline{0}}$$

$$C_k = \frac{1}{4}\int_0^4 x(t)e^{-jk\pi/2 t}\,dt \quad \leftarrow \text{calculate straight up } (k = \pm1, \pm2, \ldots)$$

$$= 1/4\int_0^2 1\, e^{-jk\pi/2 t}\,dt + 1/4\int_2^4 (-1)e^{-jk\pi/2 t}\,dt$$

$$= 1/4\ \frac{e^{-jk\pi/2 t}}{-jk\pi/2}\Bigg|_0^2 + 1/4\ \frac{e^{-jk\pi/2 t}}{+jk\pi/2}\Bigg|_2^4$$

$$= \frac{1}{-jk2\pi}\left[e^{-jk\pi} - 1\right] + \frac{1}{jk2\pi}\left[e^{-jk2\pi} - e^{-jk\pi}\right]$$

$$= \frac{2}{-jk2\pi}e^{-jk\pi} + \frac{1}{jk2\pi}\left(1 + e^{-jk2\pi}\right)$$

$$= \frac{1}{-jk\pi}\left(\cos(k\pi) - j\underbrace{\sin(k\pi)}_{\to 0}\right) + \frac{1}{jk2\pi}\left(1 + \cos(k2\pi) + j\underbrace{\sin(1)}_{\to 0}\right)$$

(1)

$$= \frac{\cos(k\pi)}{-jk\pi} + \frac{1}{jk\pi}$$

ex. cont.

For $C_k = \frac{\cos(k\pi)}{-jk\pi} + \frac{1}{jk\pi}$

If $k = \pm 2, \pm 4, \ldots$, $C_k = \frac{1}{-jk\pi} + \frac{1}{jk\pi} = 0$

If $k = \pm 1, \pm 3, \ldots$, $C_k = \frac{-1}{jk\pi} + \frac{1}{jk\pi} = \frac{2}{jk\pi} = \frac{-j2}{k\pi}$

$= \frac{2}{k\pi} e^{-j\pi/2}$

Overall, $C_k = \begin{cases} 0 & k = 0, \pm 2, \pm 4, \ldots \\ \frac{2}{jk\pi} & k = \pm 1, \pm 3, \pm 5, \ldots \end{cases}$ ← Only odd harmonics

Amplitude Spectra $|C_k| = \begin{cases} 0 & k = 0, \pm 2, \pm 4, \ldots \\ \frac{2}{|k|\pi} & k = \pm 1, \pm 3, \pm 5, \ldots \end{cases}$

Phase spectra $\measuredangle C_k = \begin{cases} 0 & k = 0, \pm 2, \pm 4, \ldots \\ -\pi/2 & k = 1, 3, 5, \ldots \\ \pi/2 & k = -1, -3, -5, \ldots \end{cases}$

Writing out the Fourier Series -

$$x(t) = \sum_{\substack{k=-\infty \\ (k \text{ odd})}}^{\infty} \frac{2}{jk\pi} e^{jk\omega_0 t} \qquad -\infty < t < \infty \qquad \omega_0 = \pi/2 \text{ rad/s}$$

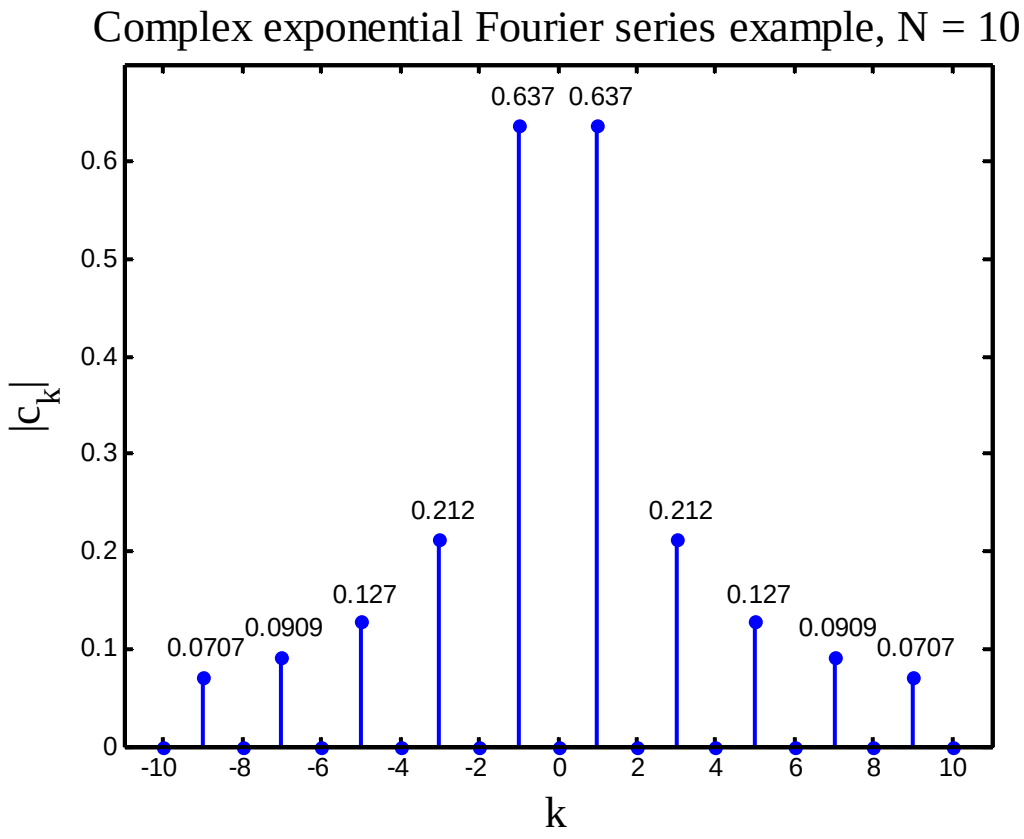
or

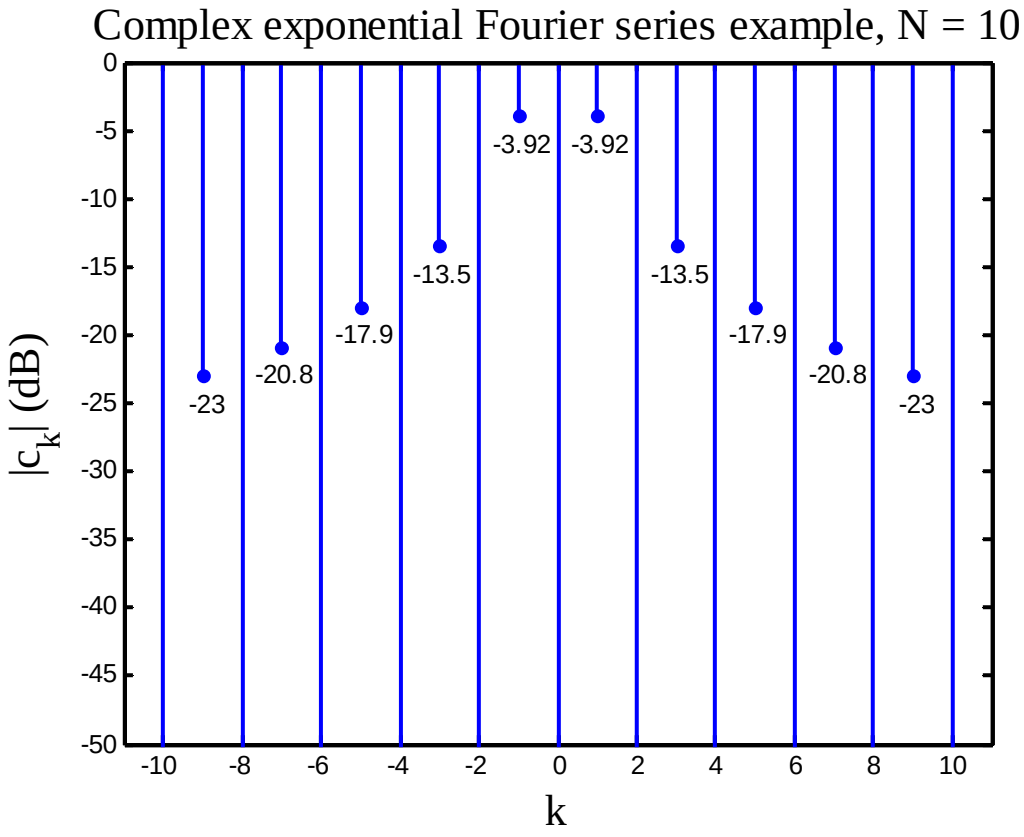
$$x(t) = \sum_{k=1}^{\infty} \frac{4}{k\pi} \cos(k\omega_0 t - \pi/2) \qquad -\infty < t < \infty \qquad \omega_0 = \pi/2 \text{ rad/s}$$

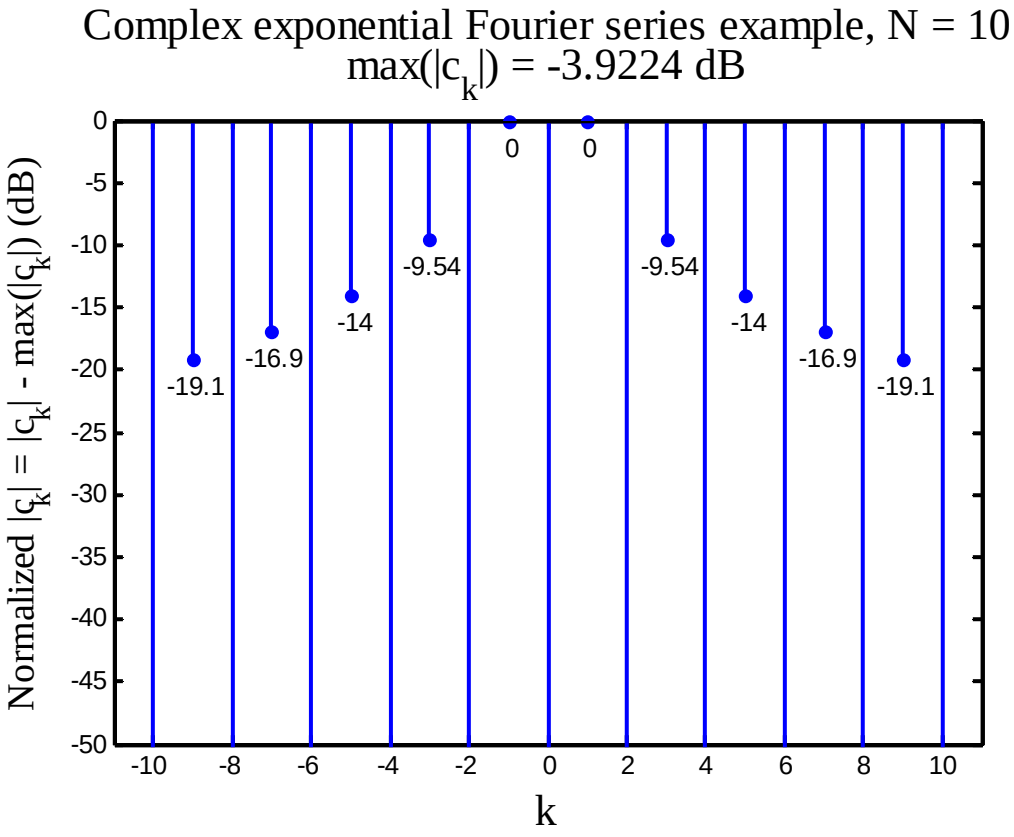
↖ used Euler Identities or $2C_k = A_k e^{j\theta_k}$

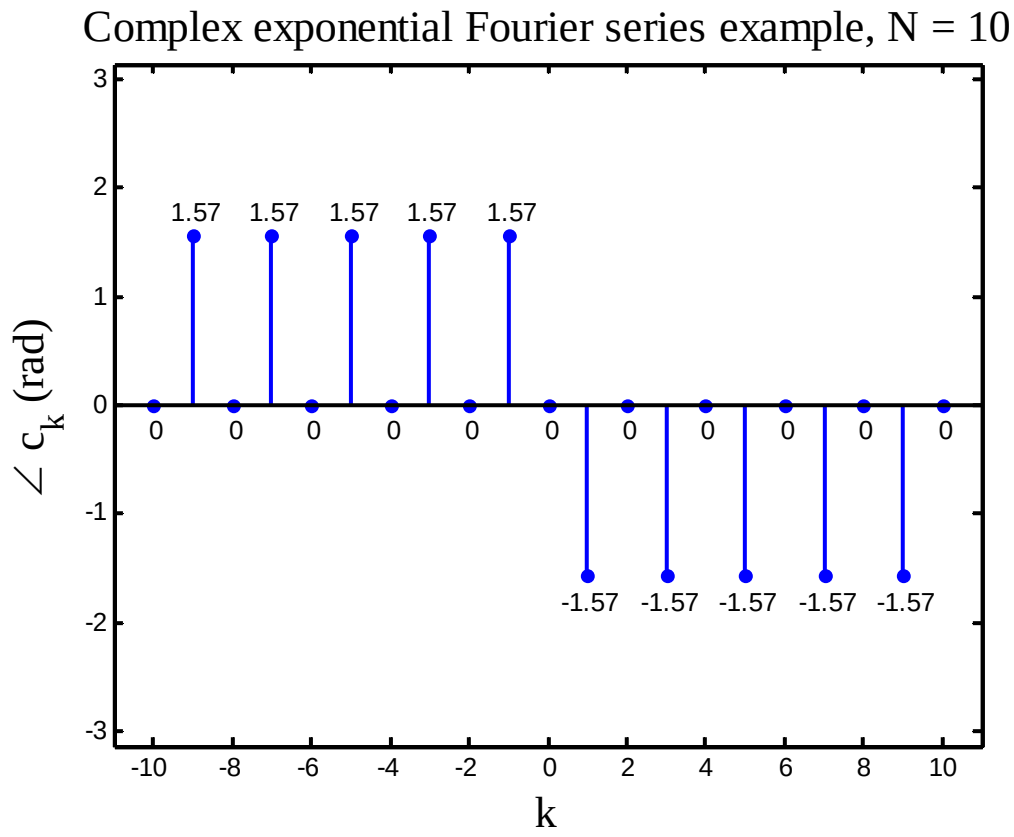
```
% Chapter 3 Fourier series spectra example
% (chap3_fourier_series2_spectra_complex.m)
%
% Computes amplitude and phase spectra for
% complex exponential Fourier series for the
% binary sine wave example
%
close all;clc;clear;
N = input('Number of harmonics ');
for k1 = 1:1:2*N+1,
    ktmp = k1-(N+1);
    if(ktmp == 0),
        ck(k1) = 0;
    else
        ck(k1) = cos(ktmp*pi)/(-1j*ktmp*pi)+1/(1j*ktmp*pi);
    end
end
k = -N:1:N;
ckmag = abs(ck);
ckang = angle(ck);
% ck terms related to mag. of voltage so 20log10() appropriate
for k3 = 1:1:2*N+1,
    if(ckmag(k3)== 0),
        ckmagdB(k3) = -100;
    else
        ckmagdB(k3) = 20*log10(ckmag(k3));
    end
end
ckmagdBnorm = ckmagdB-max(ckmagdB); % normalize to max ampl.
%
stem(k,ckmag,'b.'), axis([-(N+1) N+1 0 0.7]),
title(['Complex exponential Fourier series example, N = ',...
    num2str(N)],'fontsize',16,'fontname','times')
xlabel('k','fontsize',16,'fontname','times'),
ylabel('|c_k|','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,  %
    if(ckmag(n)>0.0009),
        text(k(n),ckmag(n)+0.045,[' ' num2str(ckmag(n),3)],...
      'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure
stem(k,ckmagdB,'b.'), axis([-(N+1) N+1 -50 0]),
title(['Complex exponential Fourier series example, N = ',...
    num2str(N)],'fontsize',16,'fontname','times')
```

```
xlabel('k','fontsize',16,'fontname','times'),
ylabel('|c_k| (dB)','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,
    if(ckang(n)>-50),
        text(k(n),ckmagdB(n)-1,[' ' num2str(ckmagdB(n),3)],...
        'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure
stem(k,ckmagdBnorm,'b.'), axis([-(N+1) N+1 -50 0]),
title({['Complex exponential Fourier series example, N = ',...
    num2str(N)];['max(|c_k|) = ',num2str(max(ckmagdB)),' dB']},...
    'fontsize',16,'fontname','times')
xlabel('k','fontsize',16,'fontname','times'),
ylabel('Normalized |c_k| = |c_k| - max(|c_k|) (dB)',...
    'fontsize',14,'fontname','times'),
% Label stems
for n=1:2*N+1,
    if(ckang(n)>-50),
        text(k(n),ckmagdBnorm(n)-1,[' ' num2str(ckmagdBnorm(n),3)],...
        'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure
stem(k,ckang,'b.'), axis([-(N+1) N+1 -pi pi]),
title(['Complex exponential Fourier series example, N = ',...
    num2str(N)],'fontsize',16,'fontname','times')
xlabel('k','fontsize',16,'fontname','times'),
ylabel('\angle c_k (rad)','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,  %
    if(ckang(n)>0.1),
        text(k(n),ckang(n)+0.35,[' ' num2str(ckang(n),3)],...
            'HorizontalAlignment','center','VerticalAlignment','top')
    else
        text(k(n),ckang(n)-0.1,[' ' num2str(ckang(n),3)],...
            'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',14)
set(findobj('type','axes'),'linewidth',2)
```

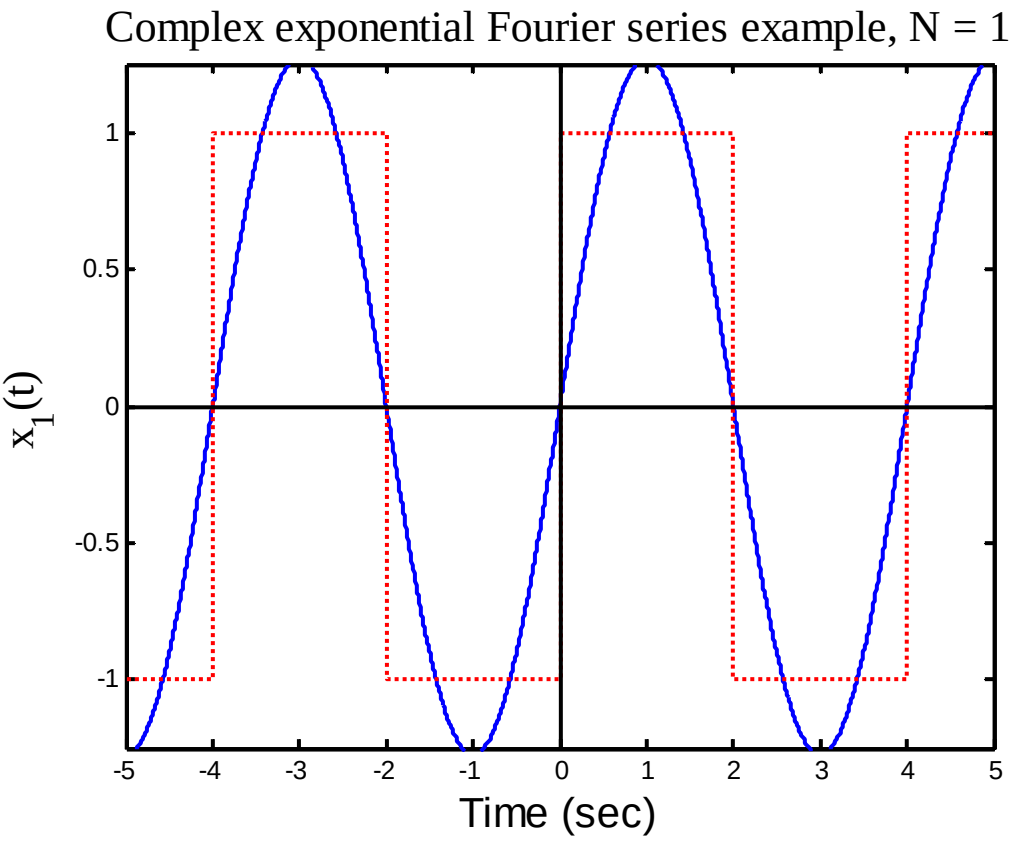

Complex exponential Fourier series example, N = 10
|c_k|
k
0.637
0.637
0.212
0.212
0.127
0.127
0.0909
0.0909
0.0707
0.0707
0
0.1
0.2
0.3
0.4
0.5
0.6
-10
-8
-6
-4
-2
0
2
4
6
8
10

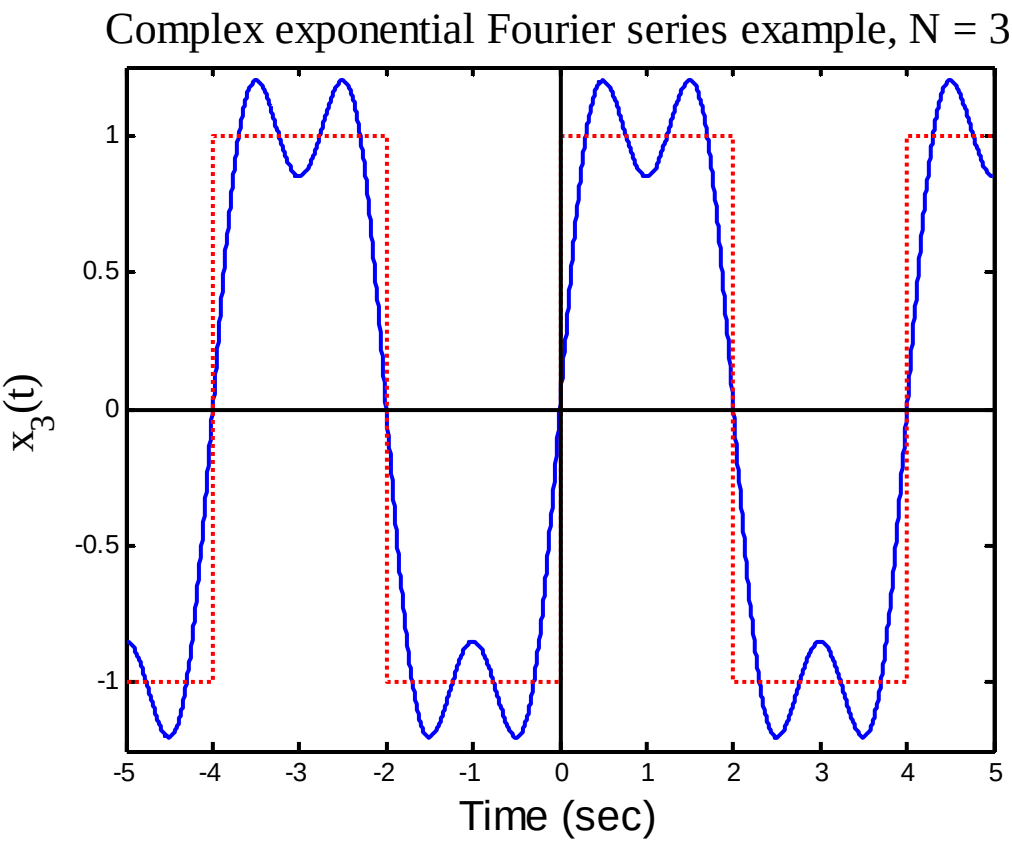

Complex exponential Fourier series example, N = 10
|c_k| (dB)
k
-3.92
-3.92
-13.5
-13.5
-17.9
-17.9
-20.8
-20.8
-23
-23
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-10
-8
-6
-4
-2
0
2
4
6
8
10

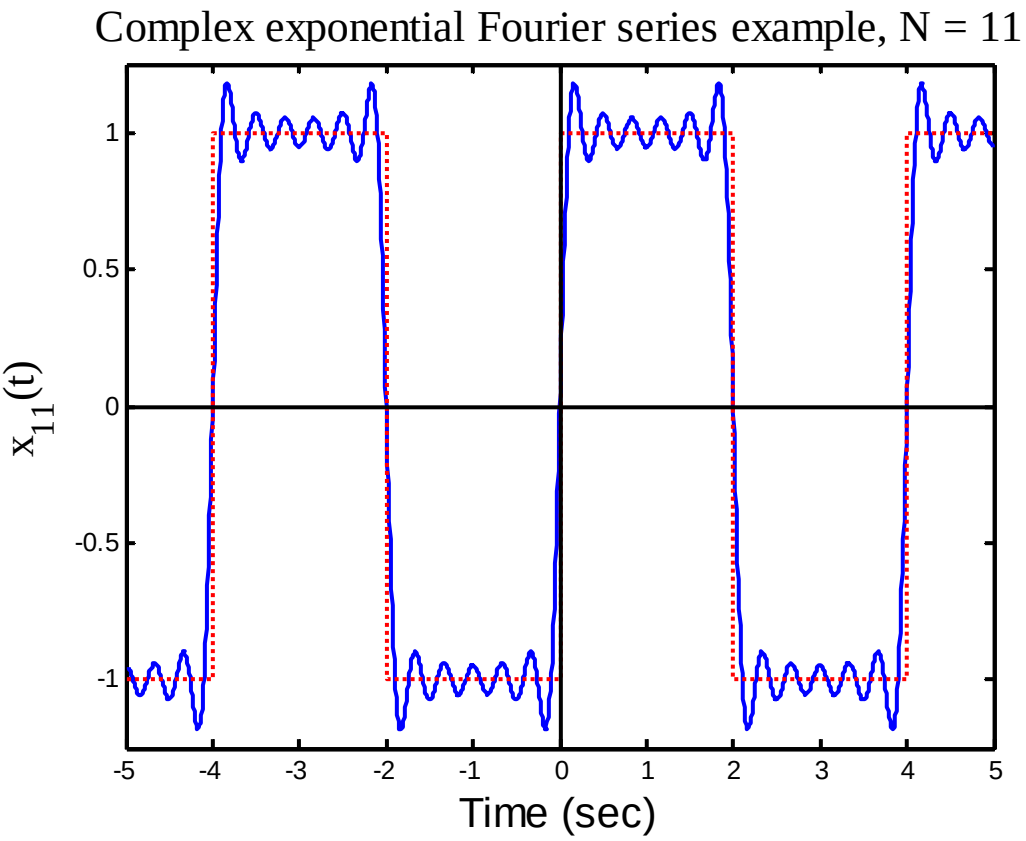

Complex exponential Fourier series example, N = 10
max(|c_k|) = -3.9224 dB
Normalized |c_k| = |c_k| - max(|c_k|) (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-19.1
-16.9
-14
-9.54
0
0
-9.54
-14
-16.9
-19.1
-10
-8
-6
-4
-2
0
2
4
6
8
10
k

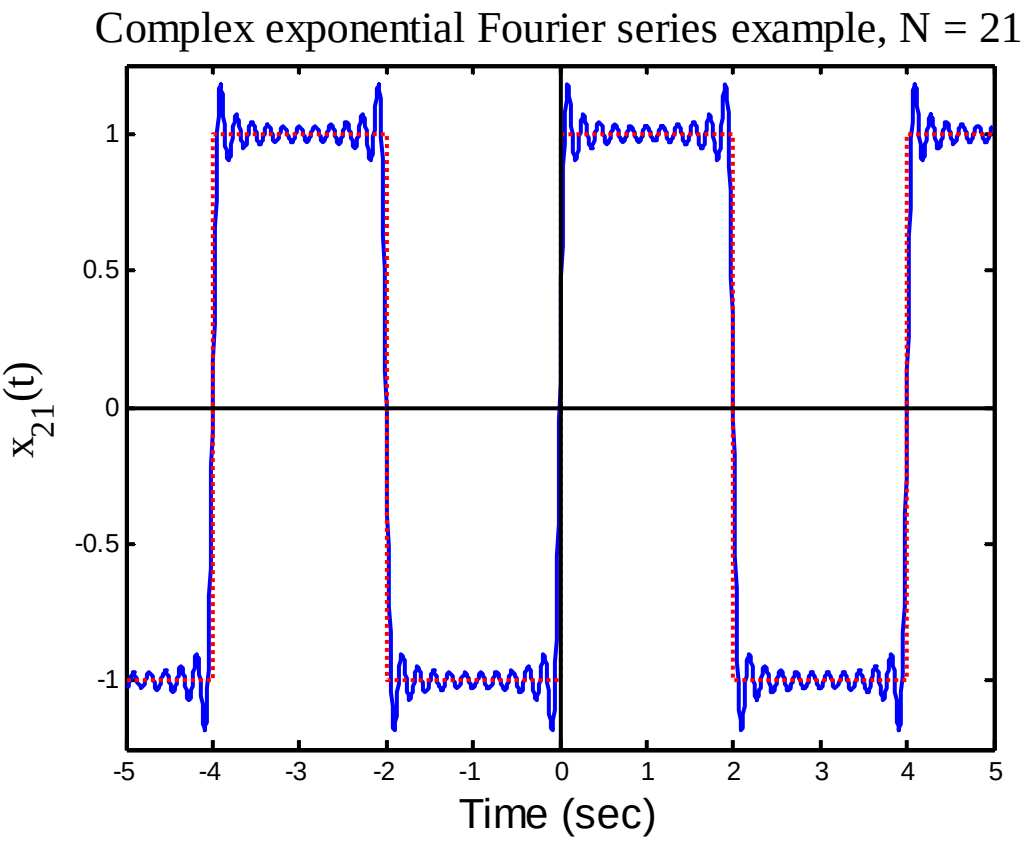

Complex exponential Fourier series example, N = 10
∠ c_k (rad)
3
2
1
0
-1
-2
-3
1.57
1.57
1.57
1.57
1.57
0
0
0
0
0
0
0
0
0
0
0
0
-1.57
-1.57
-1.57
-1.57
-1.57
-10
-8
-6
-4
-2
0
2
4
6
8
10
k

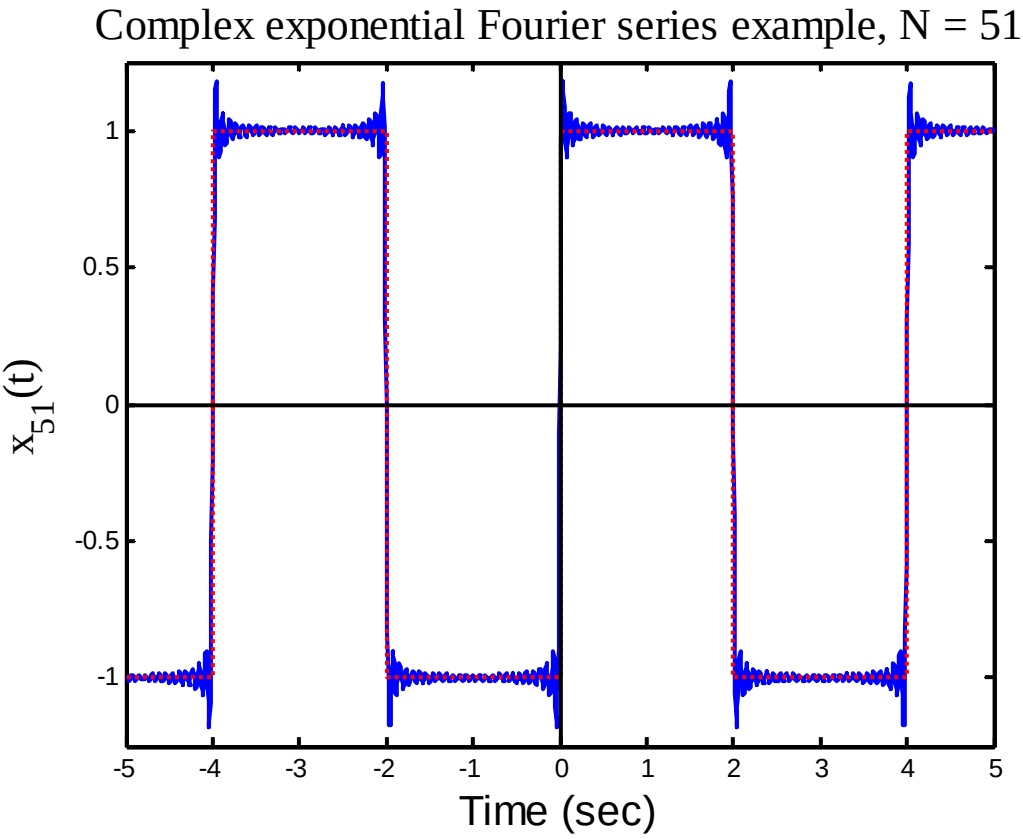
```
% Chapter 4 Fourier series example
% (chap4_fourier_series2_complex.m)
%
% Compute complex exponential Fourier series for
% binary sine wave example.
%
close all; clear; clc;
% Data points for exact waveform plot
t2=[-5,-4,-4,-2,-2,0,0,2,2,4,4,5];
x2=[-1,-1,1,1,-1,-1,1,1,-1,-1,1,1];
%
t = -5:6/1000:5;
N = input('Number of harmonics ');
w0 = pi/2;
c0 = 0;
xN = c0*ones(1,length(t));   % dc component
for k=1:2:N,                 % only need to add odd harmonics
  xN = xN+2/(1j*k*pi)*exp(1j*k*w0*t)+2/(-1j*k*pi)*exp(-1j*k*w0*t);
end
plot(t,xN,t2,x2,'r:',[-5,5],[0,0],'k-',[0,0],[-1.25,1.25],'k-'),
axis([-5 5 -1.25 1.25]),
title(['Complex exponential Fourier series example, N = ',...
    num2str(N)],'fontsize',16,'fontname','times')
xlabel('Time (sec)','fontsize',16,'fontname','times roman')
ylabel(['x_{',[num2str(N)],'}(t)'],'fontsize',16,'fontname','times')
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',18)
set(findobj('type','axes'),'linewidth',2)
```

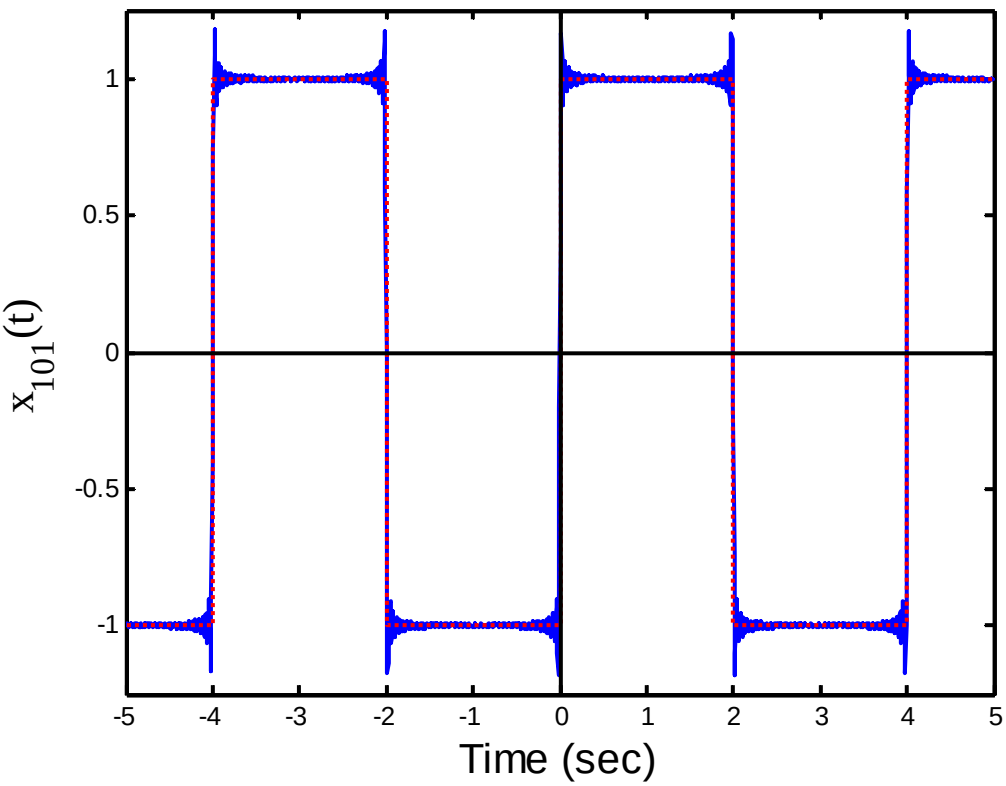

Complex exponential Fourier series example, N = 1
x1(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Complex exponential Fourier series example, N = 3
x3(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Complex exponential Fourier series example, N = 11
x11(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Complex exponential Fourier series example, N = 21
x21(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Complex exponential Fourier series example, N = 51
x51(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)


Complex exponential Fourier series example, N = 101
x101(t)
1
0.5
0
-0.5
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
Time (sec)