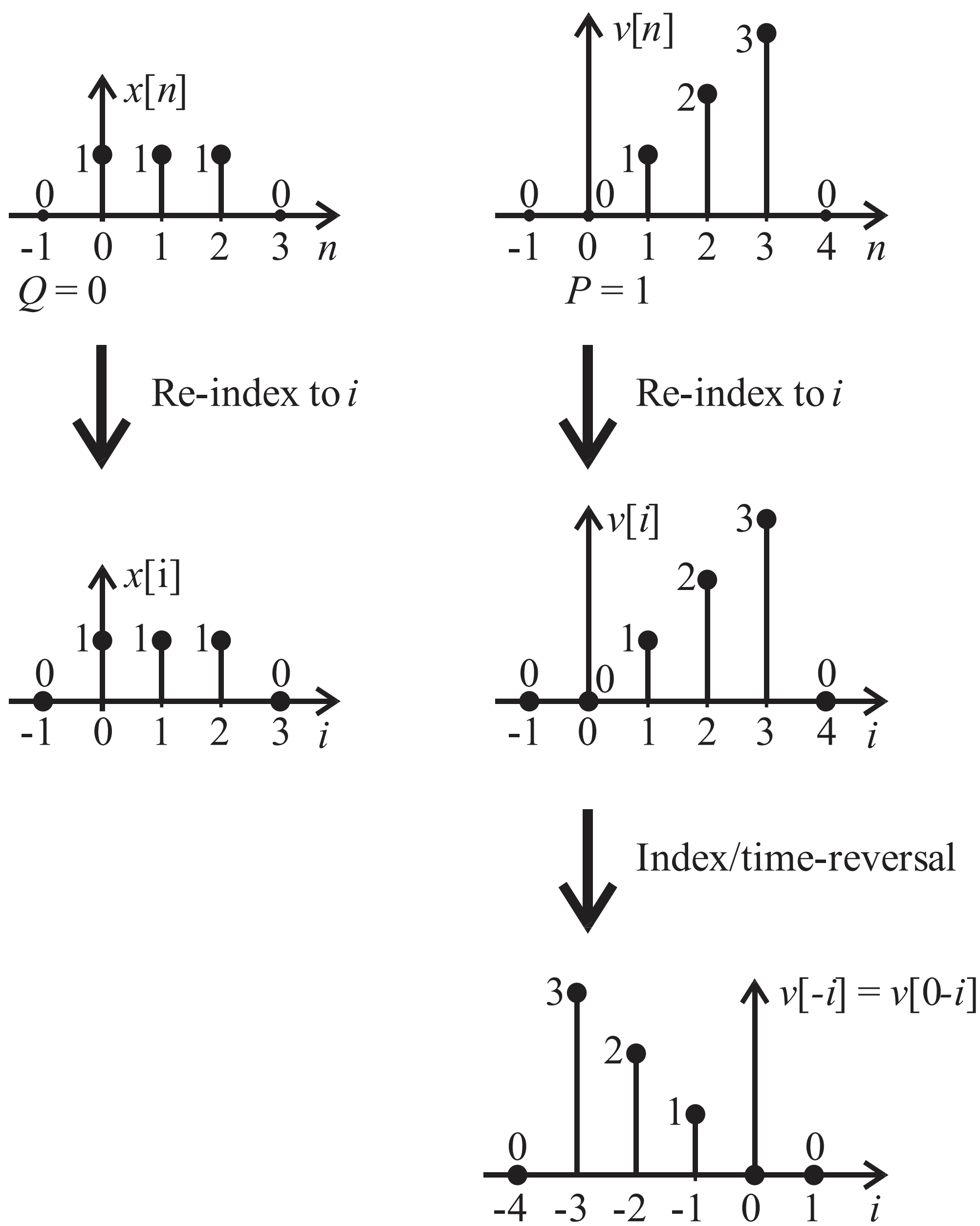
Ex. For the signals $x[n]$ and $v[n]$ shown below, manually convolve to find

$$y[n] = x[n] * v[n] = \sum_{i=-\infty}^{\infty} x[i]v[n-i],$$

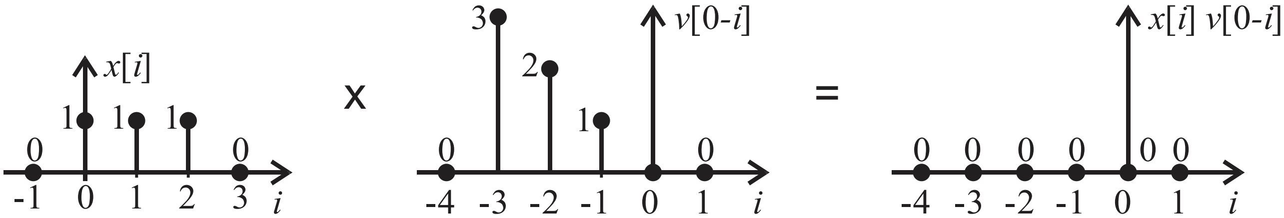
or, if $x[n]$ starts at index P and $v[n]$ starts at index Q

$$y[n] = x[n] * v[n] = \sum_{i=-\infty}^{\infty} x[i]v[n-i] = \begin{cases} 0 & n < P+Q \\ \sum_{i=Q}^{n-P} x[i]v[n-i] & n \geq P+Q \end{cases}.$$



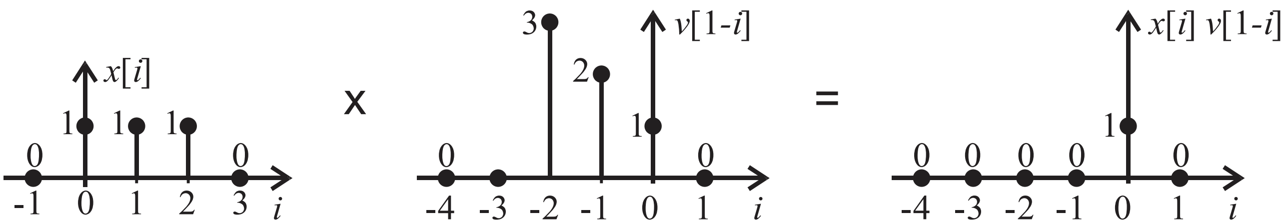
Begin convolution process-

@ $n = 0$, multiply $x[i]$ with $v[0 - i]$



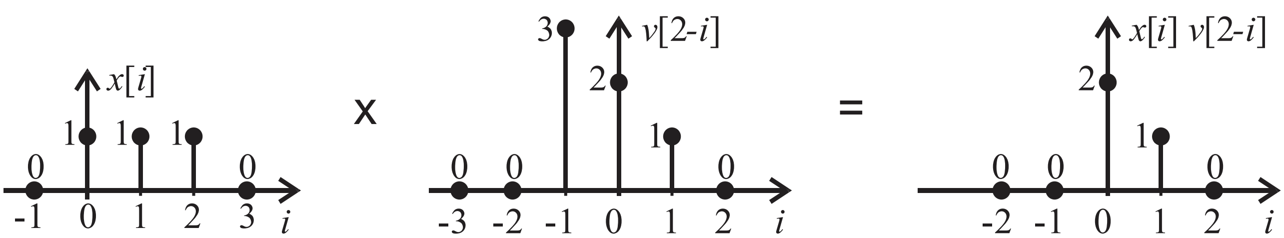
Summing the $x[i]\ v[0 - i]$ terms gives $y[0] = \ldots 0 + 0 \ldots$ $\Rightarrow$ $\boldsymbol{y[0] = 0}$

@ $n = 1$, multiply $x[i]$ with $v[1 - i]$



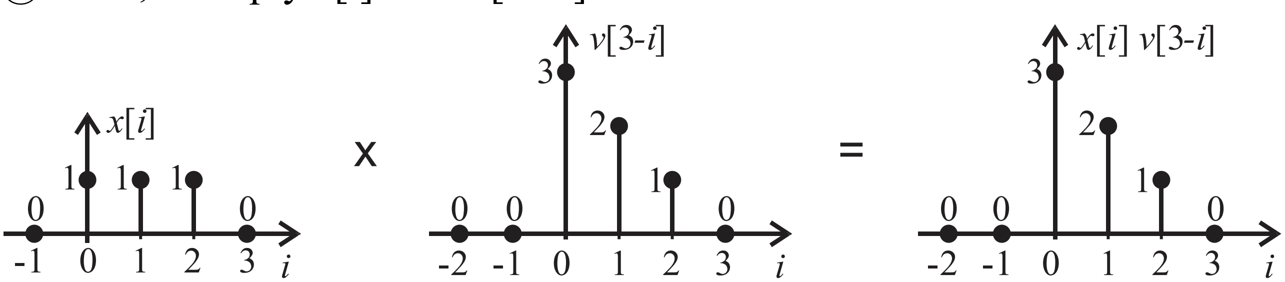
Summing the $x[i]\ v[1 - i]$ terms gives $y[1] = \ldots 0 + 1 + 0 \ldots$ $\Rightarrow$ $\boldsymbol{y[1] = 1}$

@ $n = 2$, multiply $x[i]$ with $v[2 - i]$



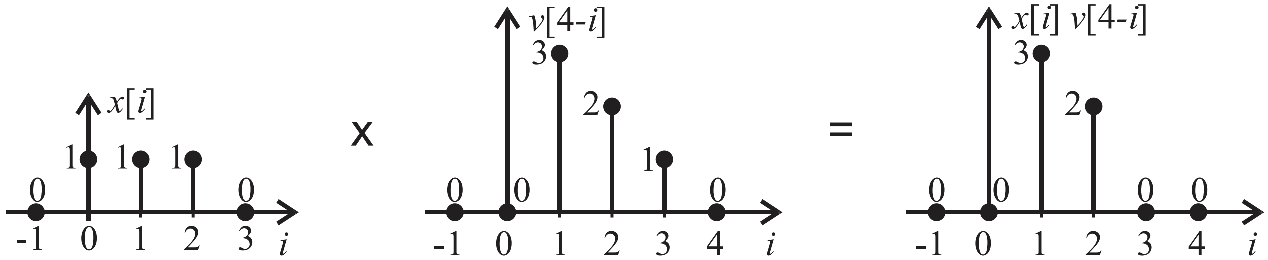
Summing the $x[i]\ v[2 - i]$ terms gives $y[2] = \ldots 0 + 2 + 1 + 0 \ldots$ $\Rightarrow$ $\boldsymbol{y[2] = 3}$

@ $n = 3$, multiply $x[i]$ with $v[3 - i]$



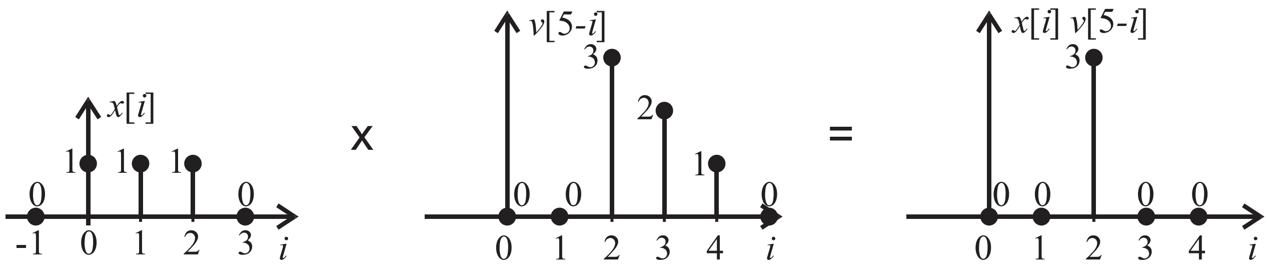
Summing the $x[i]\ v[3 - i]$ terms gives $y[3] = \ldots 0 + 3 + 2 + 1 + 0 \ldots$ $\Rightarrow$ $\boldsymbol{y[3] = 6}$

@ $n = 4$, multiply $x[i]$ with $v[4 - i]$



Summing the $x[i]\, v[4 - i]$ terms gives $y[4] = \ldots 0 + 3 + 2 + 0 \ldots \quad \Rightarrow \quad \mathbf{y[4] = 5}$

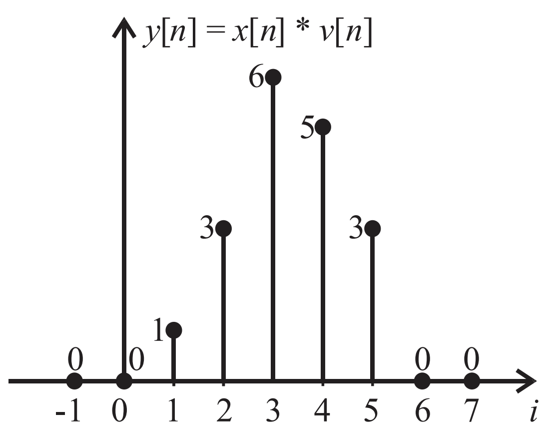
@ $n = 5$, multiply $x[i]$ with $v[5 - i]$



Summing the $x[i]\, v[5 - i]$ terms gives $y[5] = \ldots 0 + 3 + 0 \ldots \quad \Rightarrow \quad \mathbf{y[5] = 3}$

For $n \geq 6$, the signals no longer overlap. Therefore, summing the $x[i]\, v[n - i]$ terms gives $y[n \geq 6] = \ldots 0 + 0 \ldots \quad \Rightarrow \quad \mathbf{y[n \geq 6] = 0}$

Overall, we get



Note:

- First non-zero term of $y[n]$ occurs at index $n = P + Q = 0 + 1 = \mathbf{1}$
- The length of $y[n]$ is {length($x[n]$) + length($v[n]$) – 1} = 3 + 3 – 1 = **5**
- Last non-zero term of $y[n]$ occurs at $n = P + Q$ + length($y[n]$) - 1 = 0 + 1 + 5 - 1 = **5**