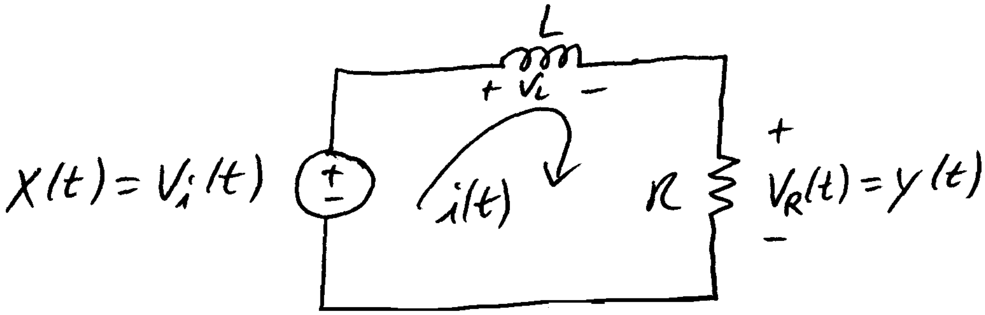
ex. Impulse Response – Series RL Circuit



To get h(t), the impulse response, we'll first determine the unit step response g(t).

From Circuits I, $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$ $t \geq 0$
for a step input $\tau = L/R$

* Since $V_i(t) = u(t) = 0$ for $t < 0$, $i(0^-) = 0$
For inductors, the current can not change instantly. So, $i(0) = i(0^-) = \underline{0}$

* As $t \to \infty$, the circuit is in a steady-state.
Therefore, $V_L(t) = L\frac{di}{dt} = 0 \Rightarrow i(t \to \infty) = \frac{V_i(t)}{R} = \underline{\frac{1}{R}}$

So, $i(t) = \frac{1}{R} + \left[0 - \frac{1}{R}\right] e^{-t/(L/R)} = \frac{1}{R} - \frac{1}{R} e^{-t/(L/R)}$ $t \geq 0$

By Ohm's Law, $V_R(t) = i(t)R = 1 - e^{-t/(L/R)}$ $t \geq 0$

For a step input, $x(t) = u(t)$ and $y(t) = V_R(t) = g(t)$

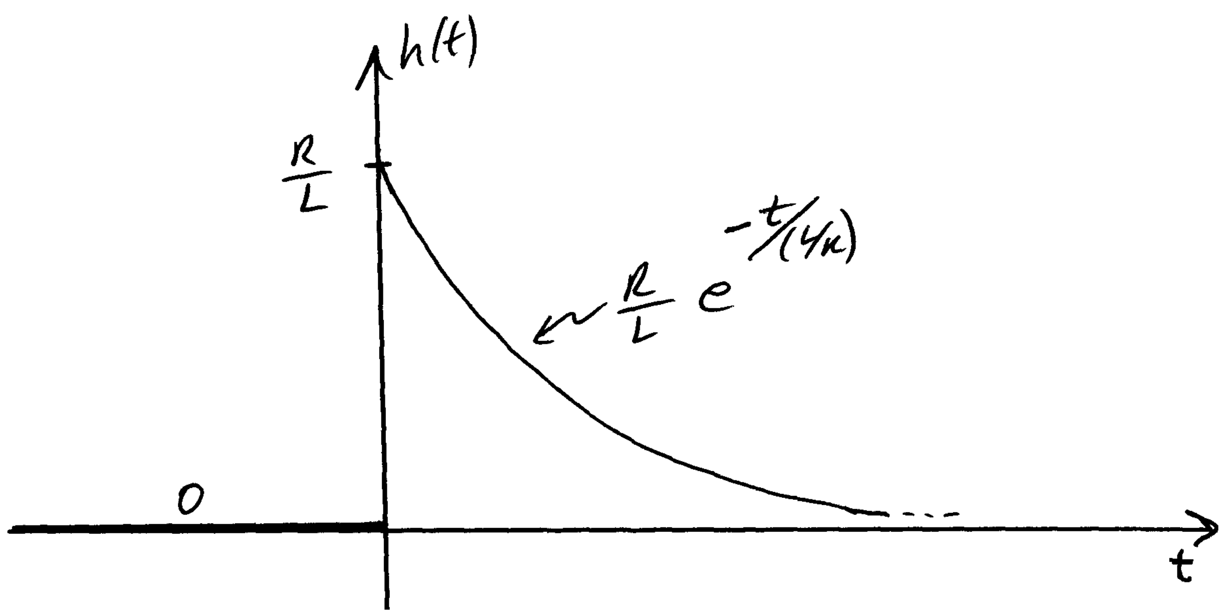
ex. cont.

$$g(t) = 1 - e^{-t/(L/R)} \quad (V) \quad t \geq 0$$
$$= 0 \quad t < 0$$

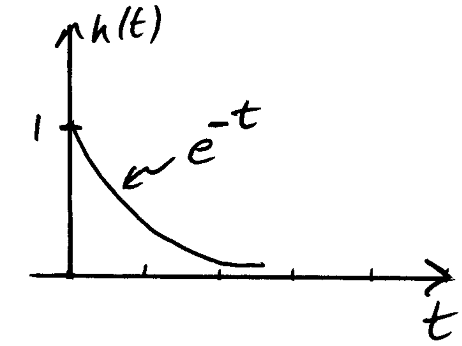
Using the derivative property of convolution

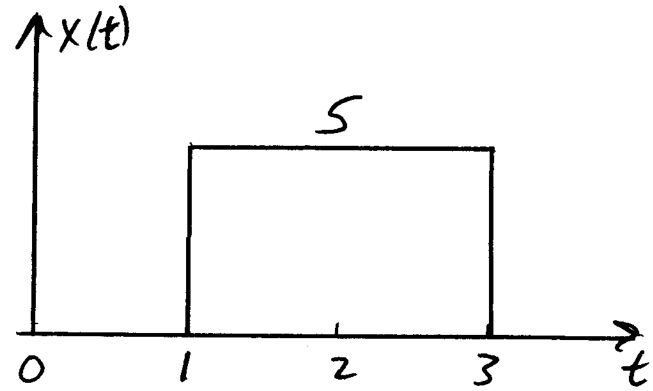
$$h(t) = \frac{dg(t)}{dt} = \frac{d\{1 - e^{-t/(L/R)}\}}{dt} = 0 - e^{-t/(L/R)}\left(\frac{-1}{(L/R)}\right)$$

$$\boxed{h(t) = \begin{cases} R/L \, e^{-t/(L/R)} & (V/s) \quad t \geq 0 \\ 0 & t < 0 \end{cases}}$$

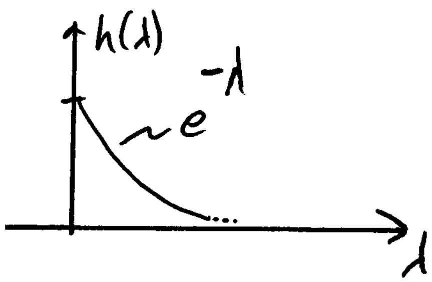


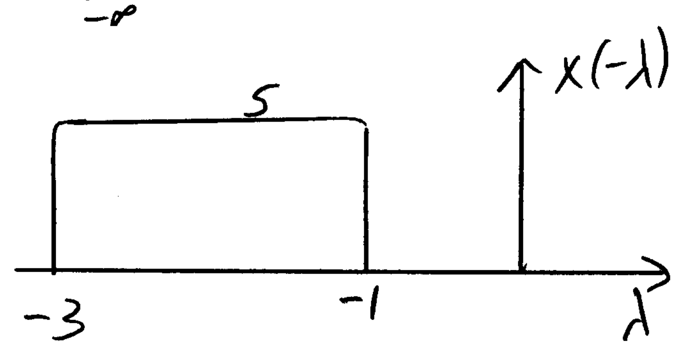
Find the output of the series RL circuit to the input $x(t) = 5p_2(t-2)$ V when $R = 1\Omega$ and $L = 1H$.



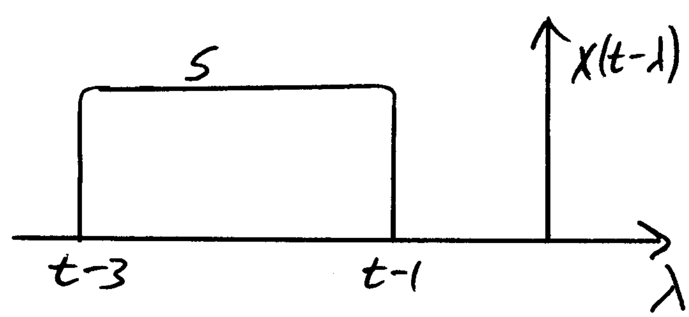


$$y(t) = x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$





↓ Time-shift by t



<u>Time Interval $t < 1s$</u>

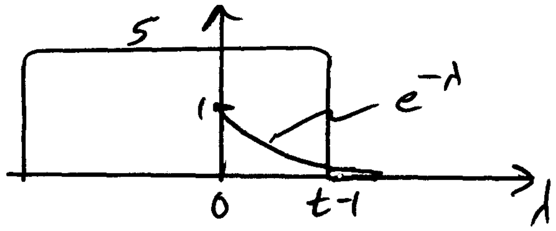
$h(\lambda)$ & $x(t-\lambda)$ do <u>Not</u> overlap, i.e., $h(\lambda)\,x(t-\lambda) = 0$

$$y(t<1s) = \int_{-\infty}^{\infty} 0\, d\lambda = \underline{\underline{0}}$$

ex. cont.

Time Interval $1 \le t < 3s$

As shown, the leading edge of $x(t-\lambda)$ overlaps $h(\lambda)$



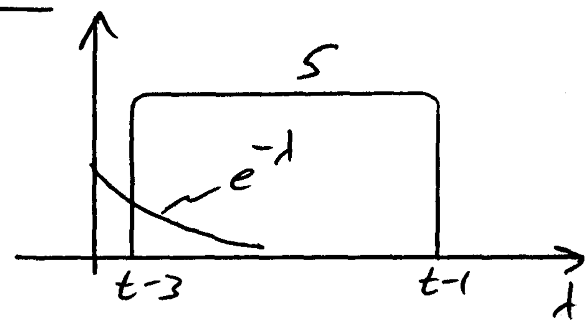
$$y(1 \le t < 3) = \int_0^{t-1} e^{-\lambda}(5)\,d\lambda = -5e^{-\lambda}\Big|_0^{t-1}$$

$$= -5e^{-(t-1)} + 5e^0 = -5e^{-t} + 5$$

$$\underline{y(1 \le t < 3s) = 5\left[1 - e^{-(t-1)}\right]}$$

Time Interval $t \ge 3s$

The signals fully overlap

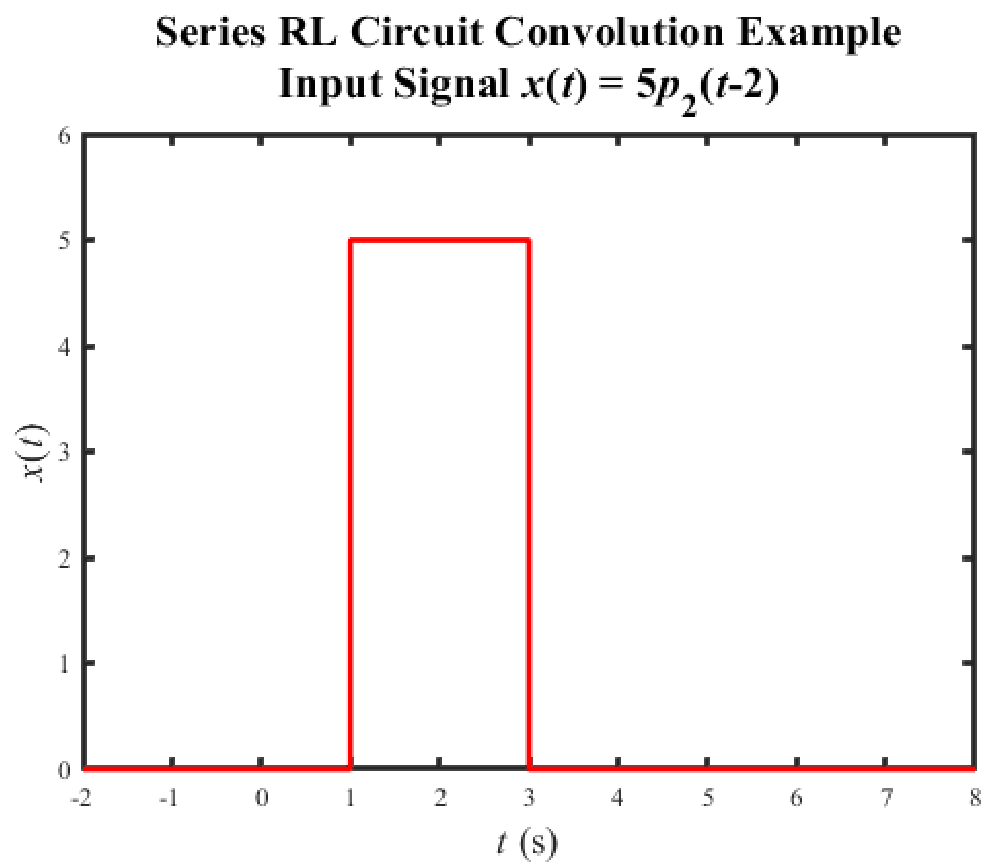


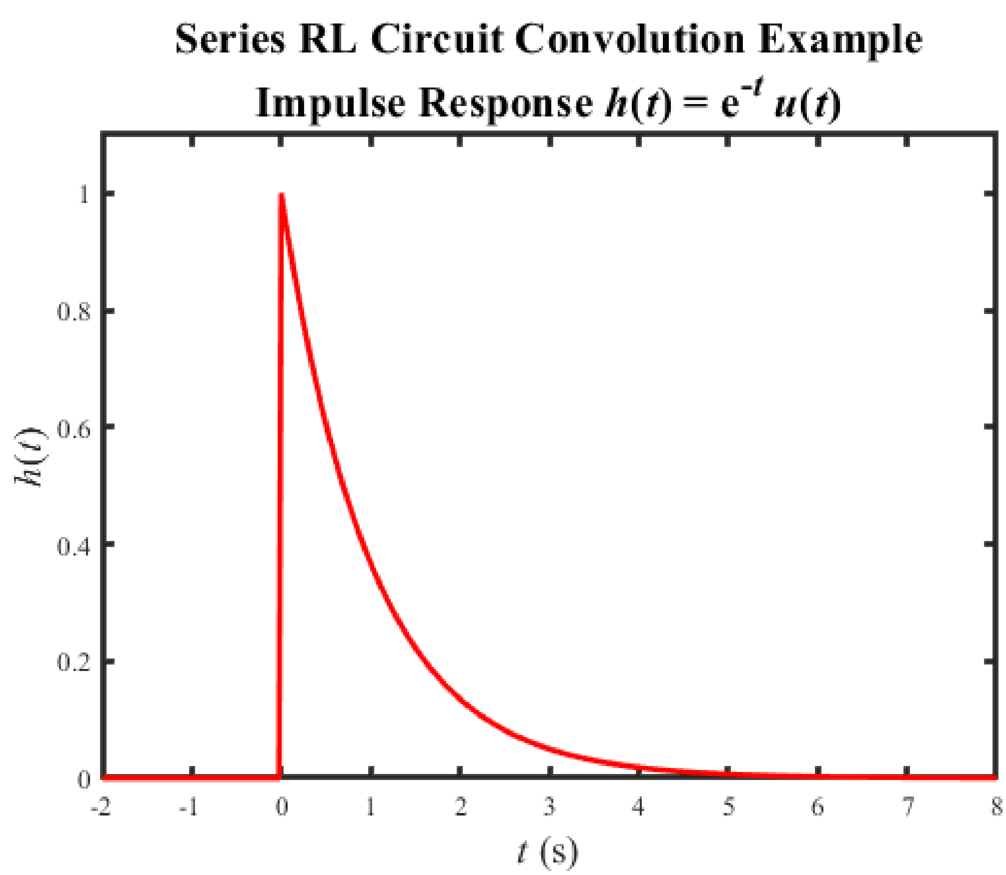
$$y(t \ge 3s) = \int_{t-3}^{t-1} e^{-\lambda}(5)\,d\lambda = -5e^{-\lambda}\Big|_{t-3}^{t-1}$$

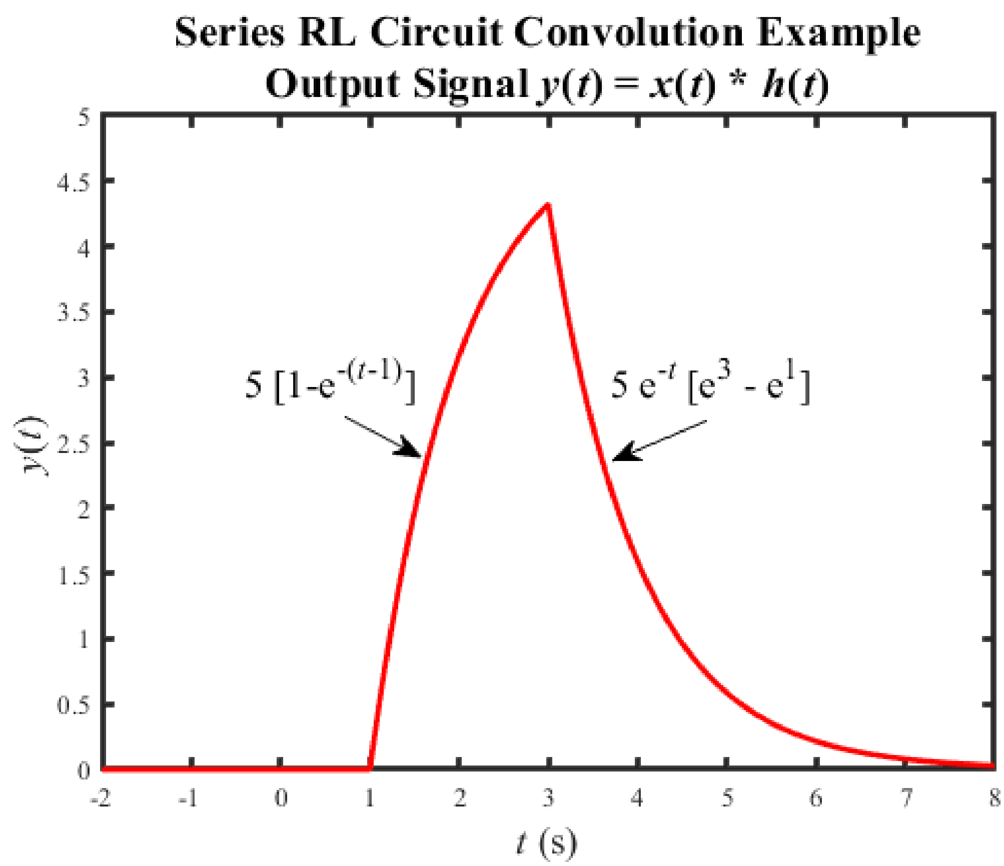
$$= -5e^{-(t-1)} + 5e^{-(t-3)} = -5e^{-t}e^{1} + 5e^{-t}e^{3}$$

$$\underline{y(t \ge 3s) = 5e^{-t}\left[e^{3} - e^{1}\right]}$$

```
% chap2_series_RL_convolution_response.m
% CT Convolution Example
% Plot the result of the CT convolution
% y(t) = x(t) * h(t)  where
%    x(t) = 5p2(t-2)  and
%    v(t) = e^(-t/tau)u(t) where tau = R/L
%
clear; clc; close all;
R = 1; L = 1; tau = R/L;
t0 = -2:0.025:1; t1 = 1:0.025:3; t2 = 3:0.025:8;
x0 = 0*t0; x1 = 5*ones(1,length(t1)); x2 = 0*t2;
y0 = 0*t0; y1 = 5*(1-exp(-t1+1)); y2 = 5*exp(-t2)*(exp(3)-exp(1));
t = [t0 t1 t2]; x = [x0 x1 x2]; y = [y0 y1 y2];
for i=1:length(t),
    if(t(i)<0),
        h(i) = 0;
    else
        h(i) = R/L*exp(-t(i)/(L/R));
    end
end
plot(t,x,'r-'), axis([-2 8 0 6]),
ylabel('{\itx}({\itt})','fontsize',14,'fontname','times'),
xlabel('{\itt} (s)','fontsize',14,'fontname','times'),
title({'Series RL Circuit Convolution Example',...
    'Input Signal {\itx}({\itt}) = 5{\itp}_2({\itt}-2)'},...
    'fontsize',16,'fontname','times'),
figure, plot(t,h,'r-'), axis([-2 8 0 1.1]),
ylabel('{\ith}({\itt})','fontsize',14,'fontname','times'),
xlabel('{\itt} (s)','fontsize',14,'fontname','times'),
title({'Series RL Circuit Convolution Example',...
    'Impulse Response {\ith}({\itt}) = e^{-{\itt} }{\itu}({\itt})'},...
    'fontsize',16,'fontname','times'),
figure,plot(t,y,'r-'), axis([-2 8 0 5]),
ylabel('{\ity}({\itt})','fontsize',14,'fontname','times'),
xlabel('{\itt} (s)','fontsize',14,'fontname','times'),
title({'Series RL Circuit Convolution Example',...
    'Output Signal {\ity}({\itt}) = {\itx}({\itt}) *
{\ith}({\itt})'},...
    'fontsize',16,'fontname','times'),
text(-0.4,3,'5 [1-e^{-({\itt}-1)}]','fontsize',14,'fontname','times')
text(3.7,3,'5 e^{-{\itt} }[e^3 -
e^1]','fontsize',14,'fontname','times')
set(findobj('type','line'),'linewidth',2,'markersize',18)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```


Series RL Circuit Convolution Example
Input Signal $x(t) = 5p_2(t\text{-}2)$
6
5
4
3
2
1
0
$x(t)$
-2
-1
0
1
2
3
4
5
6
7
8
t (s)


Series RL Circuit Convolution Example
Impulse Response $h(t) = \mathrm{e}^{-t}\, u(t)$
1
0.8
0.6
0.4
0.2
0
$h(t)$
-2
-1
0
1
2
3
4
5
6
7
8
t (s)


Series RL Circuit Convolution Example
Output Signal y(t) = x(t) * h(t)
5 [1-e^-(t-1)]
5 e^-t [e^3 - e^1]
y(t)
t (s)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-2
-1
0
1
2
3
4
5
6
7
8