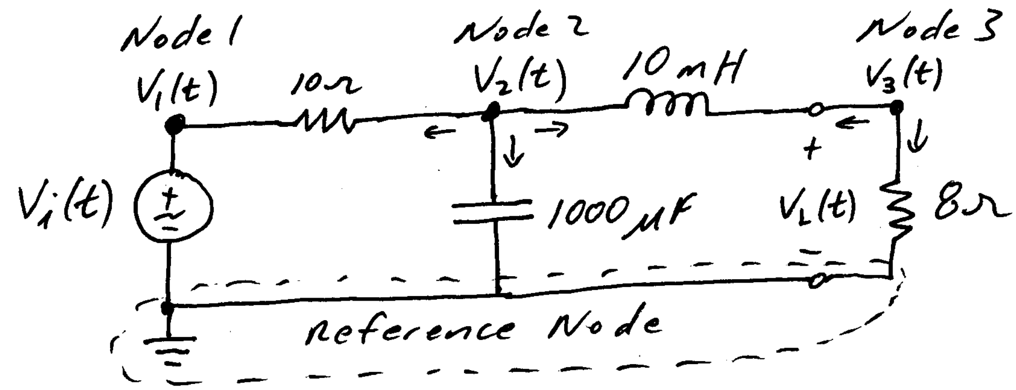
Ex. Find I/O differential equation for the lowpass filter (part of a stereo speaker crossover network) shown.



Define: input $x(t) = V_i(t)$

output $y(t) = V_L(t)$

Apply KCL / Nodal Analysis

Node 1: $V_1(t) = V_i(t) = x(t)$ (1)

Node 2: $$\frac{V_2(t) - \cancel{V_1(t)} \rightarrow x(t)}{10} + 1000 \times 10^{-6} \frac{d(V_2(t) - 0)}{dt}$$ (2)

$$+ \frac{1}{10 \times 10^{-3}} \int_{-\infty}^{t} [V_2(\lambda) - \cancel{V_3}(\lambda) \rightarrow y(\lambda)] \, d\lambda = 0$$

Node 3: $$\frac{1}{10 \times 10^{-3}} \int_{-\infty}^{t} [\cancel{V_3}(\lambda) \rightarrow y(\lambda) - V_2(\lambda)] \, d\lambda + \frac{\cancel{V_3}(t) \rightarrow y(t) - 0}{8} = 0$$ (3)

To avoid an integro-differential equation, take d/dt of equations (2) and (3).

$$0.1 \frac{dV_2}{dt} - 0.1 \frac{dx}{dt} + 0.001 \frac{d^2 V_2}{dt^2} + 100 V_2(t) - 100 y(t) = 0$$ (2)′

$$100 y(t) - 100 V_2(t) + 0.125 V_3(t) = 0$$ (3)′

Use ③′ to find V_2, $\frac{dV_2}{dt}$, & $\frac{d^2V_2}{dt^2}$ in terms of y

$$V_2 = \frac{1}{100}\left[100y + 0.125\frac{dy}{dt}\right] = y + 0.00125\frac{dy}{dt} \quad (a)$$

$$\xrightarrow{\frac{d}{dt}} \frac{dV_2}{dt} = \frac{dy}{dt} + 0.00125\frac{d^2y}{dt^2} \quad (b)$$

$$\xrightarrow{\frac{d}{dt}} \frac{d^2V_2}{dt^2} = \frac{d^2y}{dt} + 0.00125\frac{d^3y}{dt^2} \quad (c)$$

Substitute equations (a), (b), & (c) into ②′

$$0.1\left(\frac{dy}{dt} + 0.00125\frac{d^2y}{dt}\right) - 0.1\frac{dx}{dt} + 0.001\left(\frac{d^2y}{dt} + 0.00125\frac{d^3y}{dt}\right)$$
$$100\left(y + 0.00125\frac{dy}{dt}\right) - 100y = 0$$

↓ gather terms & re-arrange

$$1.25\times10^{-6}\frac{d^3y}{dt^3} + 1.125\times10^{-3}\frac{d^2y}{dt^2} + 0.225\frac{dy}{dt} = 0.1\frac{dx}{dt}$$

Since the $y(t)$ terms cancelled, integrate & divide by 1.25×10^{-6} to get

$$\underline{\underline{\frac{d^2y}{dt^2} + 900\frac{dy}{dt} + 180{,}000\,y = 80{,}000\,x}}$$

The analysis yielded a 2nd order ODE.