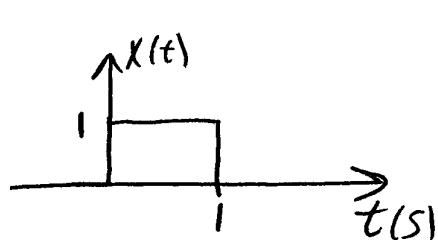
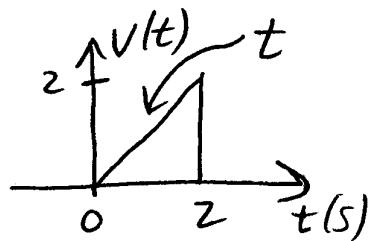


Continuous time convolution example

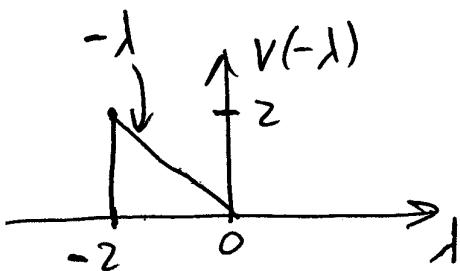
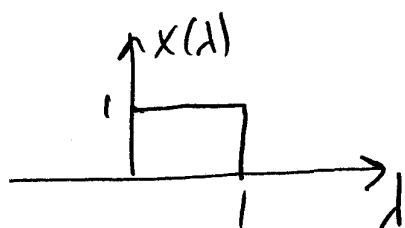
Ex. Convolve $x(t) * v(t)$



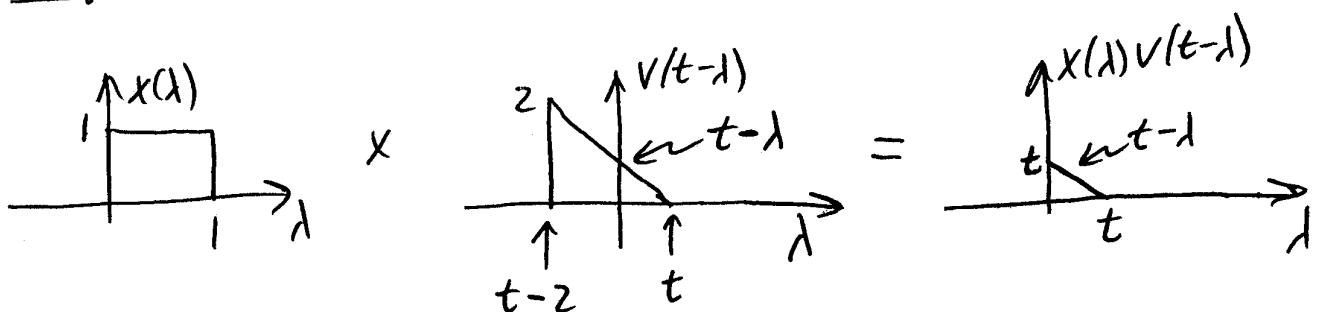
*



Step 1



Step 2 for $0 \leq t \leq 1$

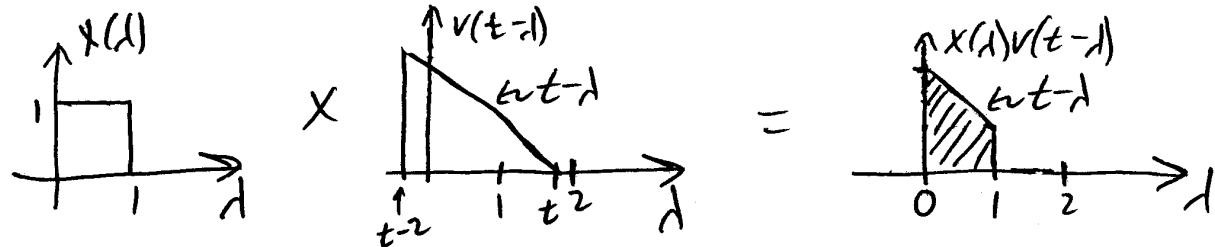


$$x(t) * v(t) = \int_0^t (t-\lambda) d\lambda = \left(t\lambda - \frac{\lambda^2}{2} \right) \Big|_0^t$$

$$= \left(t^2 - \frac{t^2}{2} \right) - (0 - 0) = \underline{\underline{\frac{t^2}{2}}} \quad 0 \leq t \leq 1$$

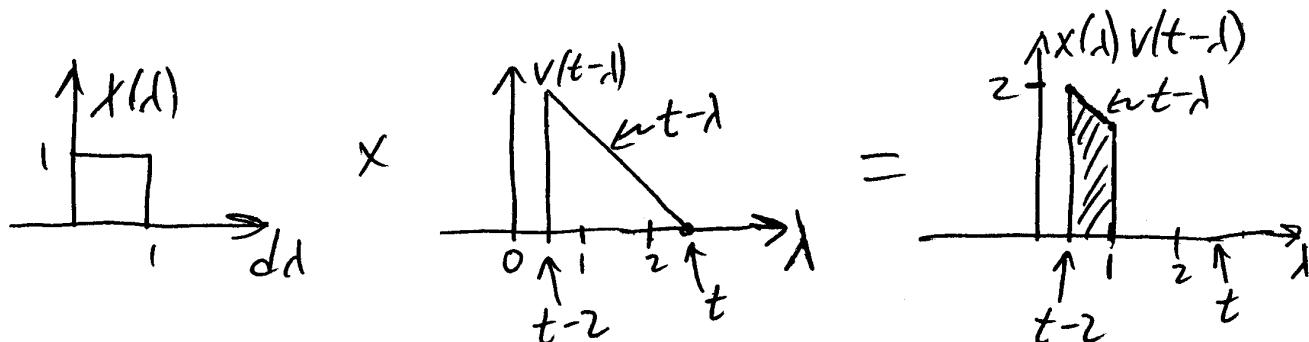
ex. cont.

Step 3 for $1 \leq t \leq 2.5$ the functions are completely overlapped



$$\begin{aligned} x(t) * v(t-\lambda) &= \int_{\lambda=0}^1 (t-\lambda) d\lambda = \left(t\lambda - \frac{\lambda^2}{2} \right) \Big|_0^1 \\ &= (t - \frac{1}{2}) - (0 - 0) = \underline{\underline{t - \frac{1}{2}}} \quad 1 \leq t \leq 2.5 \end{aligned}$$

Step 4 for $2 \leq t \leq 3.5$ we have partial overlap

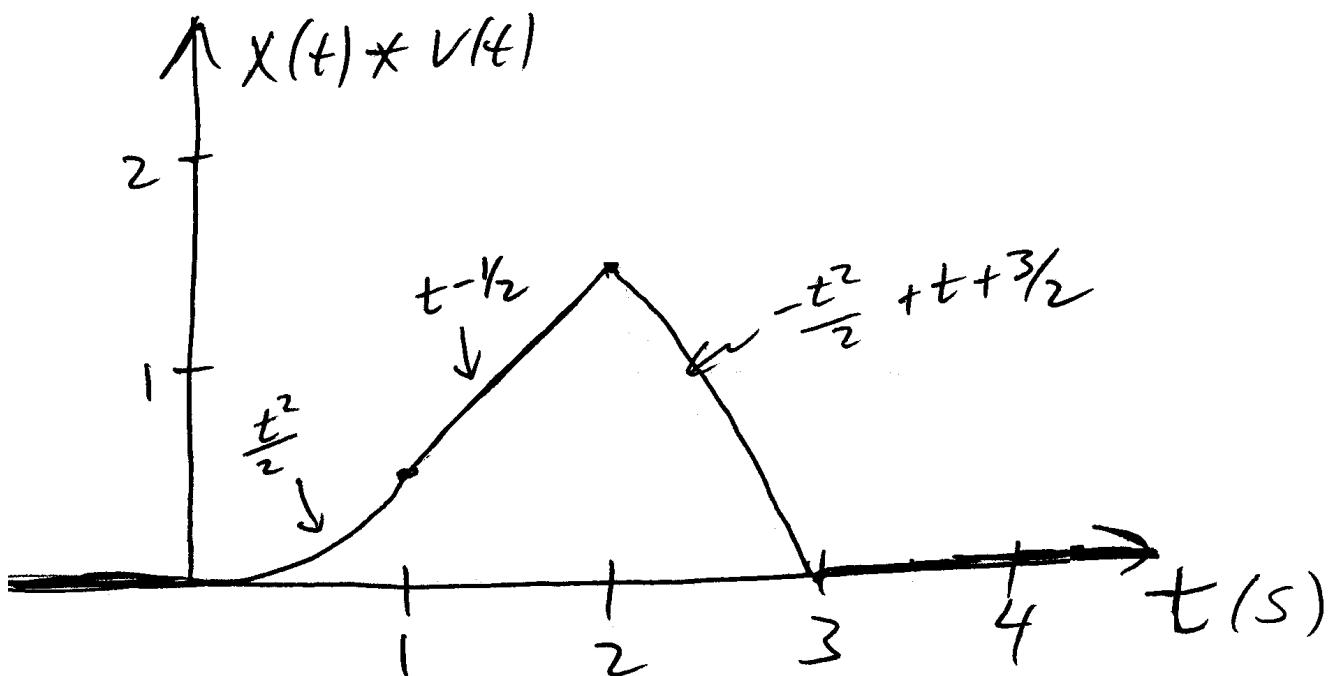


$$\begin{aligned} x(t) * v(t-\lambda) &= \int_{t-2}^1 (t-\lambda) d\lambda = \left(t\lambda - \frac{\lambda^2}{2} \right) \Big|_{t-2}^1 \\ &= \left(t - \frac{1}{2} \right) - \left(t(t-2) - \frac{(t-2)^2}{2} \right) \\ &= \underline{\underline{\frac{3}{2} - \frac{t^2}{2} + t}} \quad 2 \leq t \leq 3.5 \end{aligned}$$

Ex. cont.

Step 5 for $t > 3s$ functions do not overlap $X(t) * v(t) = 0$

$$X(t) * v(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t \leq 1s \\ t - \frac{1}{2} & 1 \leq t \leq 2s \\ -\frac{t^2}{2} + t + \frac{3}{2} & 2 \leq t \leq 3s \\ 0 & t > 3s \end{cases}$$



- Re-do plot of $x(t) * v(t)$ using MATLAB for better accuracy

```
% CT Convolution Example(chap2_CT_convolution.m)
%
% Plot the result of the CT convolution
% y(t) = x(t)*v(t) where
% x(t) = u(t)-u(t-1) and v(t) = t*(u(t)-u(t-2))
%
clear; clc; close all;
n = 0:500;
y=zeros(1,length(n));
for i=1:length(n),
    t(i)=(i-101)/100;
    if((t(i)<0)|(t(i)>=3)),
        y(i) = 0;
    end
    if((t(i)>=0)&(t(i)<1)),
        y(i) = t(i)*t(i)/2;
    end
    if((t(i)>=1)&(t(i)<2)),
        y(i) = t(i) - 1/2;
    end
    if((t(i)>=2)&(t(i)<3)),
        y(i) = -t(i)*t(i)/2 + t(i) + 3/2;
    end
end
plot(t,y,'r-',[0 0],[0 1.6],'b:',[1 1],[0 1.6],'b:',...
    [2 2],[0 1.6],'b:',[3 3],[0 1.6],'b:');
axis([-1 4 0 1.6]),
ylabel('{\ity}({\itt})','fontsize',14,'fontname','times'),
xlabel('{\itt} (s)','fontsize',14,'fontname','times'),
title('Continuous Time Convolution Example',...
    'fontsize',16,'fontname','times'),
text(0.1,0.3,'{\itt}^2 / 2','fontsize',14,'fontname','times')
text(1.1,1.2,'{\itt} - 1/2','fontsize',14,'fontname','times')
text(2.5,1.2,'-{\itt}^2/2 + {\itt} + 3/2','fontsize',14,'fontname',...
    'times')
set(findobj('type','line'),'linewidth',2,'markersize',18)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```

