Example- Second-order ODE with step input

As previously shown, the analytic solution to the second-order ODE

$$\frac{d^2y(t)}{dt^2} + 22\frac{dy(t)}{dt} + 21\,y(t) = 21\,x(t)$$

with

$$y(0) = 0,\ \left.\frac{dy(t)}{dt}\right|_{t=0} = 0,\ \text{and}\ \ x(t) = u(t)$$

is

$$\underline{y(t) = -1.05\,e^{-1t} + 0.05\,e^{-21t} + 1 \quad t \ge 0}.$$

Solving this second-order ODE using the Runge-Kutta MATLAB solver ode45() is a bit trickier than the first-order case. In the function call, we must break the derivative down into a series of first-order derivatives.

I.e.,

$$\frac{d\,y_1(t)}{dt} = y_2(t)$$

$$\frac{d^2\,y_1(t)}{dt^2} = \frac{d\,y_2(t)}{dt} = -22\,y_2(t) - 21\,y_1(t) + 21u(t)$$

where $y_1(t) = y(t)$. This is the equivalent of

$$\frac{d^2y(t)}{dt^2} = -22\frac{d\,y(t)}{dt} - 21\,y(t) + 21\,x(t).$$

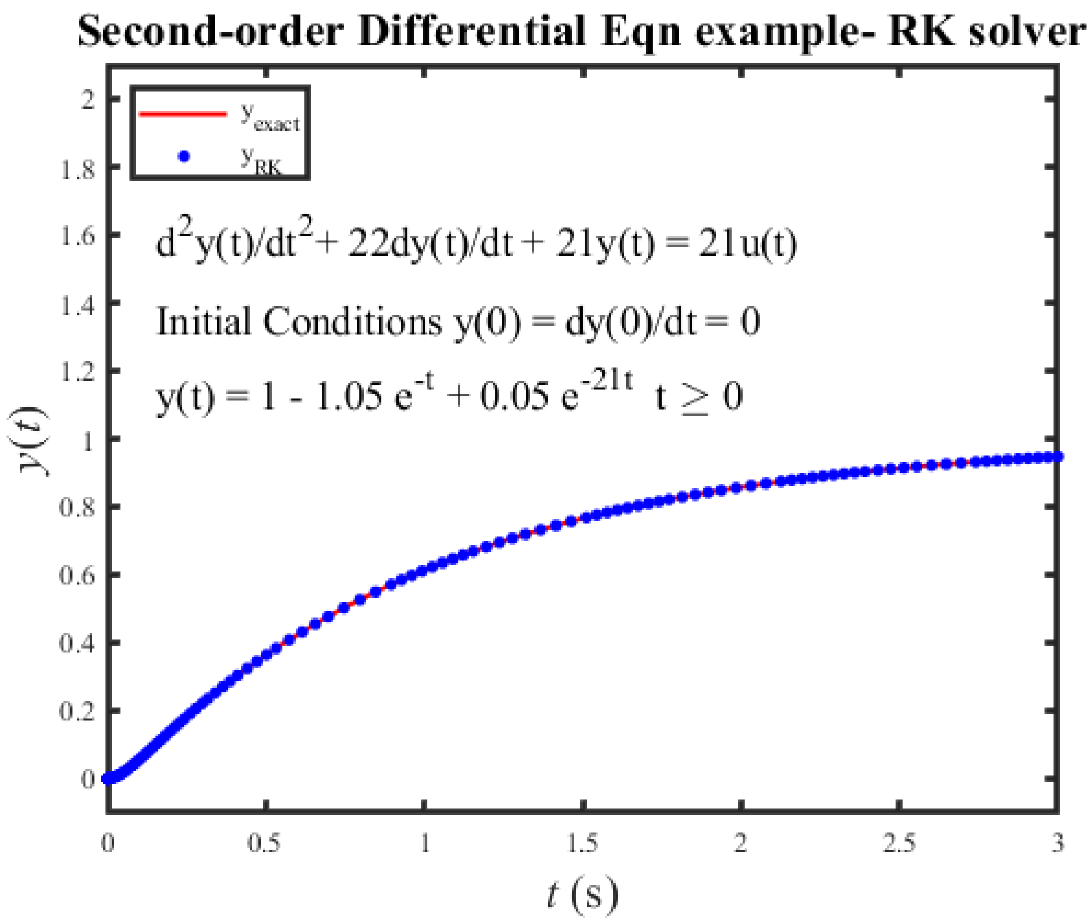
```
% Numerical ODE Solution Example (chap2_2ODE_RK_soln.m)
%
% Find approximate numerical solution to the second-order
% ordinary differential equation (ODE)
%                d2y/dt2 + 22*dy/dt + 21*y = 21*u(t),
% w/ initial conditions y(0) = y'(0) = 0,
% using an explicit Runge-Kutta (4,5) formulation
% (ode45 MATLAB command).
% Compare numerical results with exact solution.
clear;clc;close all;
% Runge-Kutta solution
tspan = [0 3];      % vector w/ initial and final times
y0 = [0,0];                % initial conditions
[trk,yrk] = ode45(@ODE_RK_example2,tspan,y0); % call a MATLAB ODE solver
% *** Analytic solution for comparison ***
texact=0:0.01:3;
yexact=1-1.05*exp(-texact)+0.05*exp(-21*texact);
%
plot(texact,yexact,'r',trk,yrk(:,1),'b.')
legend(' y_{exact}',' y_{RK}','location','NW'), axis([0 3 -0.1 2.1])
ylabel('{\ity}({\itt})','fontsize',16,'fontname','times')
xlabel('{\itt} (s)','fontsize',16,'fontname','times')
title('Second-order Differential Eqn example- RK solver','fontsize',...
    16,'fontname','times')
text(0.15,1.6,'d^{2}y(t)/dt^{2}+ 22dy(t)/dt + 21y(t) = 21u(t)',...
    'fontsize',14,'fontname','times')
text(0.15,1.35,'Initial Conditions y(0) = dy(0)/dt = 0','fontsize',...
    14,'fontname','times')
text(0.15,1.15,'y(t) = 1 - 1.05 e^{-t} + 0.05 e^{-21t}  t \geq 0',...
    'fontsize',14,'fontname','times')
set(findobj('type','line'),'linewidth',1.5,'markersize',12)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```

****

In a separate m-file, `ODE_RK_example2.m`, define the differential equation

****

```
% Differential eqn for Runge-Kutta Numerical
% ODE Solution Example (ODE_RK_example2.m)
function dy = ODE_RK_example2(t,y);
dy = zeros(2,1); %initialize 2-element vector
dy(1)= y(2);    % derivative of dy(1)/dt = y(2)
dy(2) = -22*y(2) - 21*y(1) + 21*1;
```



- Excellent agreement between numerical and analytic results.