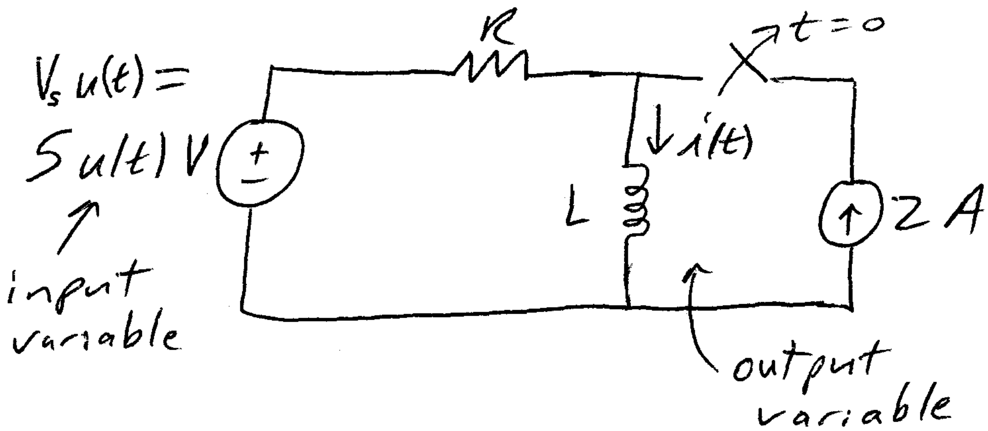
Ex. Find the first-order I/O differential equation for the RL circuit shown.



Choose $R = 10\,\Omega$ $\tau = L/R = 0.1$

$L = 1\,H$

From circuit diagram $i(0^-) = i(0) = 2A$

(Initial condition) $i(\infty) = 5V/10\,\Omega = 0.5\,A$

Diff. Eqn (applied KVL) for $t \geq 0$

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_s}{L}$$

$$\frac{di(t)}{dt} + 10\,i(t) = 5u(t) \quad t \geq 0$$

Analytic Solution $i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau} \quad t \geq 0$

$$i(t) = 0.5 + 1.5\, e^{-10t}\ A \quad t \geq 0$$

Discretize the I/O first-order ODE differential equation for the RL circuit using both the forward-difference (text) and backward-difference DT approximations.

Forward-difference (Text)

$$\frac{i[n+1]-i[n]}{T} + 10\, i[n] = 5u[n]$$

$$\hookrightarrow\; i[n+1] = (1-10T)\, i[n] + T(5u[n]) \qquad n = 0, 1, \ldots$$

Re-index $n \to n-1$

$$(1)\quad i[n] = (1-10T)\, i[n-1] + T(5u[n-1]) \qquad n = 1, 2, \ldots$$

$a_0 = 1$ (coefficient of $i[n]$), $-a_1 = (1-10T)$, $b_1 = T$, $x[n-1] = 5u[n-1]$

Initial conditions $i[0] = 2A$ + $x[0] = 5$

Backward-difference

$$\frac{i[n]-i[n-1]}{T} + 10\, i[n] = 5u[n]$$

$$\hookrightarrow\; i[n] = \left(\frac{1}{1+10T}\right) i[n-1] + \left(\frac{T}{1+10T}\right)(5u[n]) \qquad n = 1, 2, \ldots$$

$-a_1 = \frac{1}{1+10T}$, $b_0 = \frac{T}{1+10T}$, $x[n] = 5u[n]$

w/ I.C. $i[0] = 2A$ No need for I.C. for $x[n]$

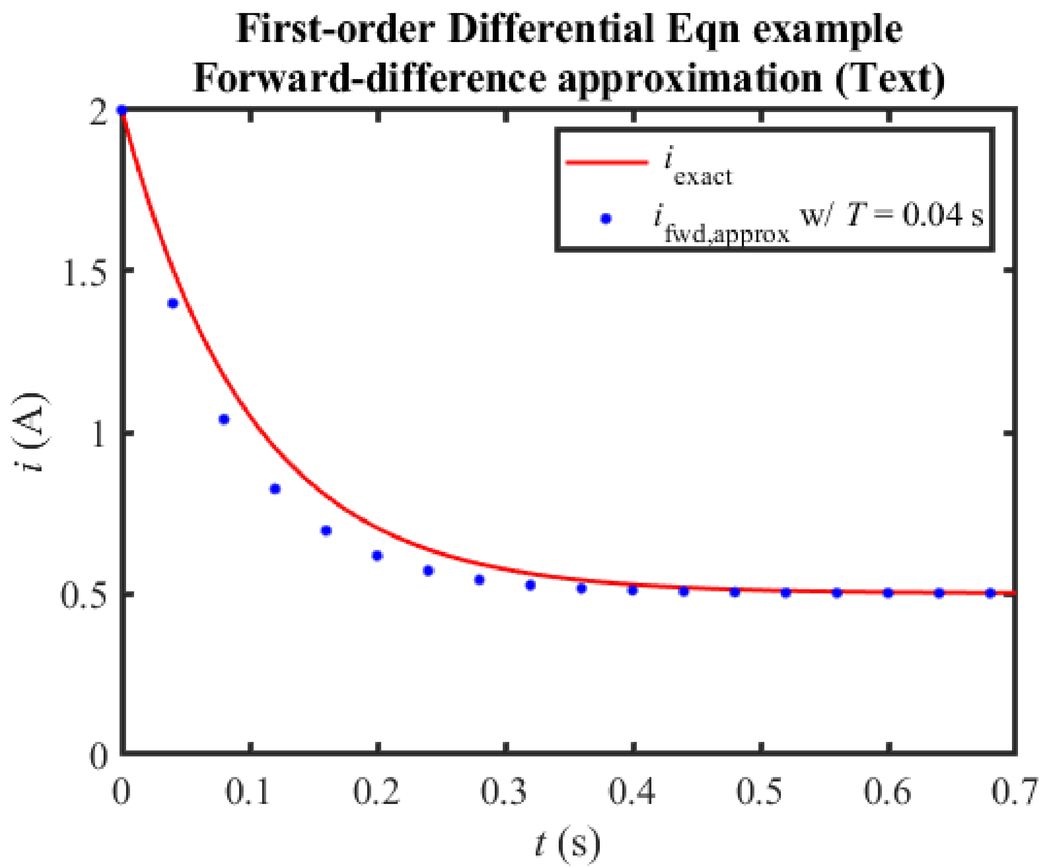
Numerical solution using forward-difference DT approximation (text) to the I/O first-order ODE differential equation.

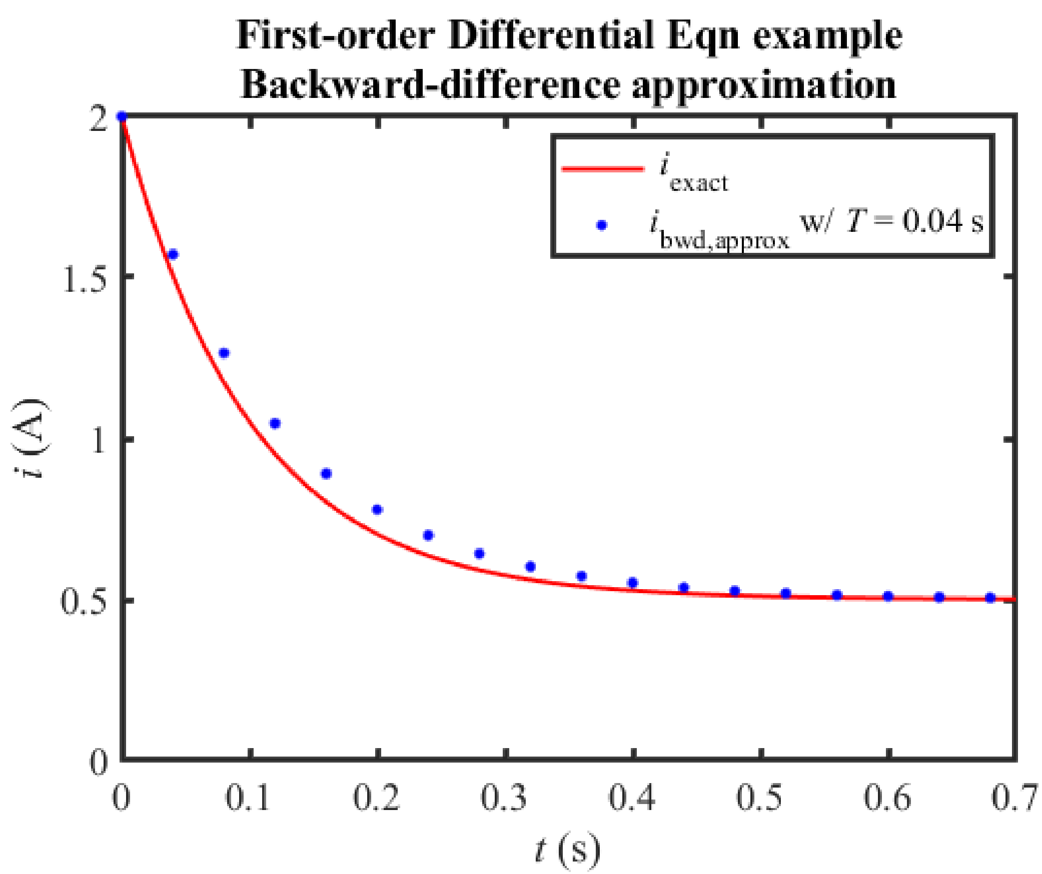
```
% Numerical ODE Solution Example (chap2_1ODE_euler_soln_fwd.m)
%
% Find approximate numerical solution to a first-order
% ordinary differential equation (ODE) by using a forward-difference
% Euler's approximation for derivatives to change
% it into a first-order difference equation which can be solved
% recursively.  Compare numerical results with exact solution.
%
close all; clear; clc;
% *** Forward-difference Euler approximation ***
T = 0.01;                    % Time step for numerical approximation
tstop = 0.7;                 % How far to go in time in seconds
a = [-1+10*T]; b=[0,T];      % Coefficient vectors for recur
n = 1:1:round(tstop/T);      % Define index vector for recur()
x = 5*ones(1,length(n));     % 5 V dc input
x0 = [5]; y0=[2];            % initial conditions at n=0 (t=nT=0)
y = recur(a,b,n,x,x0,y0);    % yields output for n=1,2,3,...
iapprox = [y0,y]; n = [0,n]; % tack on values at t=nT=0
%
t = 0:0.005:tstop;           % Define time steps for analytic sol'n
iexact = 0.5+1.5*exp(-10*t); % Analytic solution to i(t) for ODE
%
plot(t,iexact,'r',n*T,iapprox,'b.')
legend(' {\iti}_{exact}',['{\iti}_{fwd,approx} w/ {\itT} = ',...
    num2str(T),' s']), axis([0 tstop 0 2])
ylabel('{\iti} (A)','fontsize',16,'fontname','times')
xlabel('{\itt} (s)','fontsize',16,'fontname','times')
title({'First-order Differential Eqn example';...
    'Forward-difference approximation (Text)'},'fontsize',...
    16,'fontname','times')
set(findobj('type','line'),'linewidth',1.5,'markersize',12)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','axes'),'fontsize',14,'fontname','times')
```

Numerical solution using backward-difference DT approximation to the I/O first-order ODE differential equation.

```
% Numerical ODE Solution Example (chap2_1ODE_euler_soln_bwd.m)
%
% Find approximate numerical solution to a first-order
% ordinary differential equation (ODE) by using a
% backward-difference Euler's approximation for derivatives to
% change it into a first-order difference equation which can be
% solved recursively.  Compare numerical results with exact solution.
%
close all; clear; clc;
% *** Backward-difference Euler approximation ***
T = 0.01;                  % Time step for numerical approximation
tstop = 0.7;               % How far to go in time in seconds
a = [-1/(1+10*T)]; b = [T/(1+10*T)]; % Coefficient vectors for recur
n = 1:1:round(tstop/T);     % Define index vector for recur()
x = 5*ones(1,length(n));    % 5 V dc input
x0 = []; y0 = [2];          % initial conditions at n=0 (t=nT=0)
y = recur(a,b,n,x,x0,y0);   % yields output for n=1,2,3,...
iapprox = [y0,y]; n = [0,n]; % tack on values at t=nT=0
%
t = 0:0.005:tstop;          % Define time steps for analytic sol'n
iexact = 0.5+1.5*exp(-10*t); % Analytic solution to i(t) for ODE
%
plot(t,iexact,'r',n*T,iapprox,'b.')
legend(' {\iti}_{exact}',['{\iti}_{bwd,approx} w/ {\itT} = ',...
    num2str(T),' s']), axis([0 tstop 0 2])
ylabel('{\iti} (A)','fontsize',16,'fontname','times')
xlabel('{\itt} (s)','fontsize',16,'fontname','times')
title({'First-order Differential Eqn example';...
    'Backward-difference approximation'},'fontsize',...
    16,'fontname','times')
set(findobj('type','line'),'linewidth',1.5,'markersize',12)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','axes'),'fontsize',14,'fontname','times')
```

<u>Try a step size of 0.04 seconds</u>- Fair/so-so agreement w/ analytic solution





<u>Try a step size of 0.01 seconds</u>- Good agreement w/ analytic solution