EE 313 Signals and Systems Exam 2 Example

Name _____

Instructions: <u>Show all work for full credit</u>. Write answers in indicated places. Attach equation sheet to exam.

1) A discrete-time signal has a discrete Fourier transform of

$X_0 = 11.3000$,

$X_1 = 2.4180 + j\ 1.7568$,

$X_2 = 0.1820 + j\ 0.5600$,

$X_3 = 0.1820 - j\ 0.5600$, and

$X_4 = 2.4180 - j\ 1.7568$.

Determine how many points N there are in the signal. Given a sampling rate of 2.75 µs, what discrete-time Fourier transform frequency Ω_2 and continuous-time frequency ω_2 correspond to the point X_2? Then, calculate and sketch a labeled stem plot of the time domain signal $x[n]$.

$N =$ ____**5**____ $\Omega_2 =$ ___**0.8π = 2.51 rad**___ $\omega_2 =$ ____**913,918 rad/s**____

1) continued

$x[n]$

3.3

1.7

1.3

1.7

3.3

1   2   3   4 n
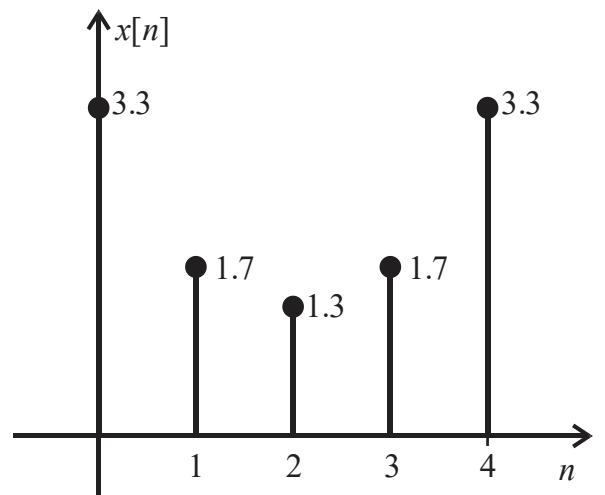
2) For the shown periodic signal, determine the fundamental period T & frequency ω_0. Is it an odd function, even function, or neither? Find the complex exponential coefficients c_0, c_1, c_2, c_{-1}, & c_{-2}.
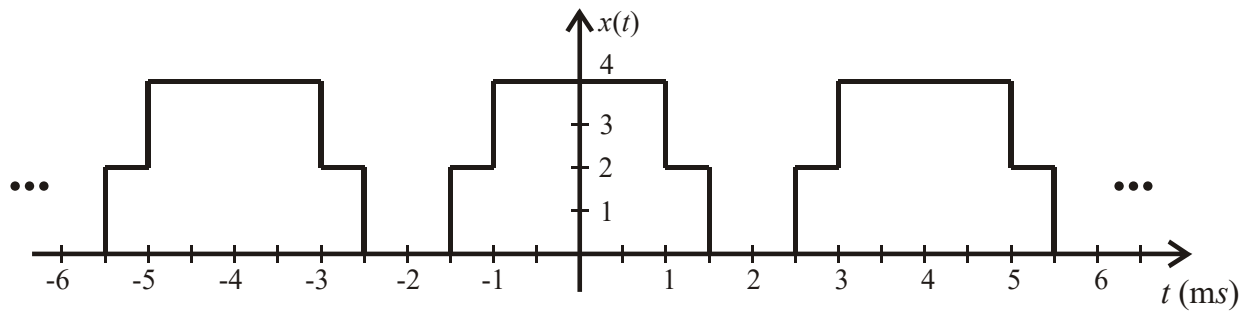
$T =$ ___**4 ms**___ $\omega_0 =$ **500π = 1570.8 rad/s**   Odd, **even**, or neither? (circle correct answer)

$c_0 =$ ___**2.5**___ $c_1 =$ ___**1.08678**___ $c_{-1} =$ ___**1.08678**___

$c_2 =$ ___**-1/ π = 0.31831**___ $c_{-2} =$ ___**-1/ π = 0.31831**___

3) Given the discrete-time signals $x[n] = 4\,\text{sgn}[n]$ and $v[n] = -2p_5[n]$, determine the discrete-time Fourier transform of the following signals.

a) $x[n]$

$$X(\Omega) = \frac{8}{1 - e^{-j\Omega}} \qquad -\infty < \Omega < \infty$$

b) $v[n]$

$$V(\Omega) = \frac{-2\sin(2.5\Omega)}{\sin(0.5\Omega)} \qquad -\infty < \Omega < \infty$$

c) $w[n] = n\,x[n]$

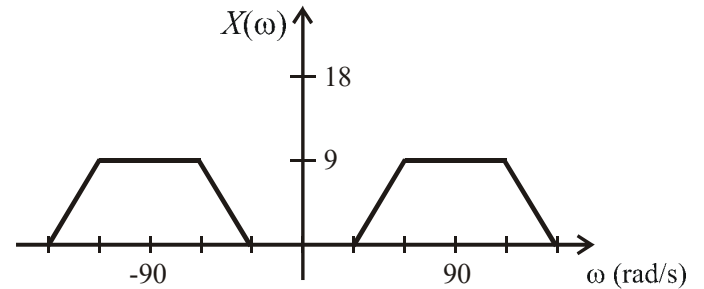$$W(\Omega) = \frac{8e^{-j\Omega}}{\left(1 - e^{-j\Omega}\right)^2} \qquad -\infty < \Omega < \infty$$

d) $y[n] = x[n-2] * v[-n]$

$$Y(\Omega) = \left(\frac{8\,e^{-j2\Omega}}{1 - e^{-j\Omega}}\right)\left(\frac{-2\sin(-2.5\Omega)}{\sin(-0.5\Omega)}\right) = \left(\frac{-16\,e^{-j2\Omega}}{1 - e^{-j\Omega}}\right)\left(\frac{\sin(2.5\Omega)}{\sin(0.5\Omega)}\right) \qquad -\infty < \Omega < \infty$$

4) Answer the following questions.

    a) Find the time-domain signal whose frequency spectrum is shown below.

$$X(\omega)$$

18

9

-90   90 ω (rad/s)

$$x(t) = \left[\frac{1080}{\pi} \text{sinc}^2 \left(\frac{30\,t}{\pi} \right) - \frac{270}{\pi} \text{sinc}^2 \left(\frac{15\,t}{\pi} \right) \right] \cos(90t) \quad -\infty < t < \infty$$

    b) Find the Fourier transform $Y(\omega)$ of $y(t) = te^{-50t}\,u(t)$.

$$Y(\omega) = \left(\frac{1}{j\omega + 50} \right)^2 \quad -\infty < \omega < \infty$$

    c) Find the Fourier transform of $v(t) = \dfrac{d^2 y(t)}{dt^2}$

$$V(\omega) = \frac{-\omega^2}{(j\omega + 50)^2} \quad -\infty < \omega < \infty$$