In the lab, the first-order *RC* circuit was set up with $v_S(t) = 8\,[u(t) - u(t - 6\text{ ms})]$ V and the known/nominal component values shown.

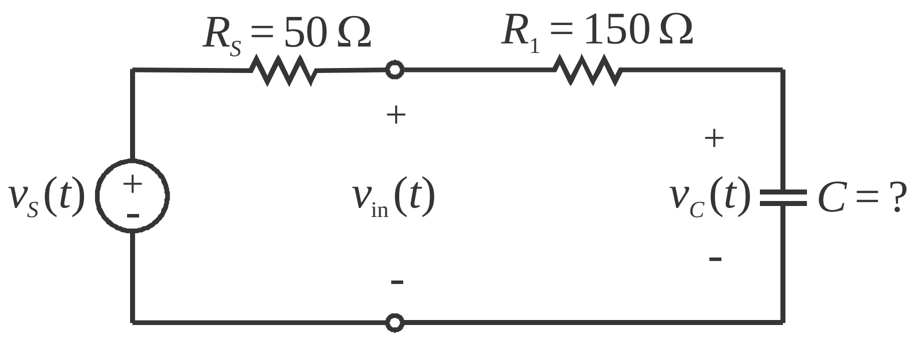
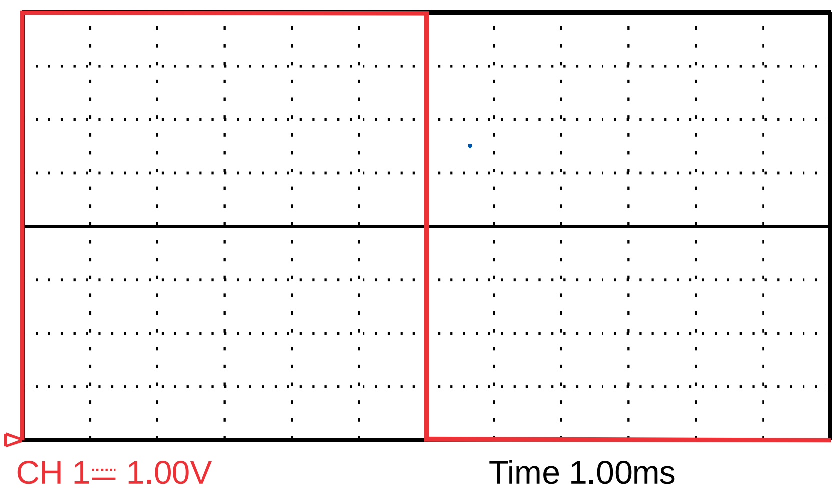


Figure 1 First-order *RC* circuit

We will use experimental measurements to determine the time-constant τ_{RC} for the capacitor voltage $v_C(t)$ waveform. This measured time-constant τ_{RC} and the known/measured resistor/resistance values will be used to determine the unknown capacitance *C*.

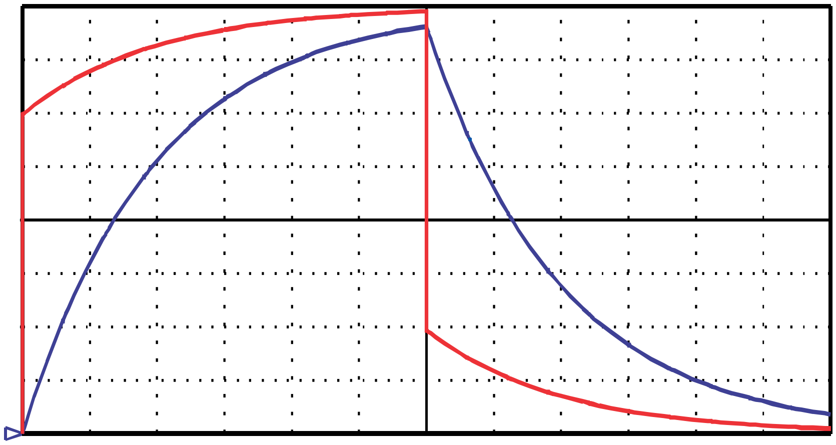
Steps

1) Assume that $\underline{\boldsymbol{R_S = 50\ \Omega}}$ as specified by function generator manual.
2) Use a digital multimeter to measure the resistance of resistor $\underline{\boldsymbol{R_1 = 146.87\ \Omega}}$.
3) Connect the function generator directly to channel 1 of the oscilloscope to set up $v_S(t)$. The screen display is shown below. [Note ‘CH 1 1.00V’ means 1 V per division in the vertical direction and ‘Time 1.00ms’ means 1 ms per division in the horizontal direction.]



Using the oscilloscope measurements menu, determine that the minimum voltage is **0.00 V**, maximum voltage is **8.00 V**, pulse width is **6.00 ms** and period is **12.00 ms**.

4) Connect circuit shown in Figure 1. Then, connect the oscilloscope to measure $v_{IN}(t)$ [CH 1- blue line] & $v_C(t)$ [CH 2- red line] with the screen display shown below. Note that $v_{IN}(t) \neq v_S(t)$ due to the voltage drop across R_S.



Using the oscilloscope cursors menu, measure the $v_C(t)$ at 1 ms intervals (i.e., where the $v_C(t)$ trace crosses the dashed vertical lines).

Table 1 Measured capacitor voltages versus time

Time (ms)	v_C (V)	Time (ms)	v_C (V)
0	0	7	4.60
1	3.15	8	2.78
2	5.12	9	1.66
3	6.24	10	0.98
4	6.96	11	0.66
5	7.44	12	0.38
6	7.68		

5) For $t \geq 6$ ms, the capacitor voltage decays in the mathematical form

$$v_C(t) = v_C(6\,\text{ms})\, e^{-(t - 0.006)/\tau} = 7.68\, e^{-(t - 0.006)/\tau} \text{ V} \quad t \geq 6 \text{ ms.}$$

Doing a time-shift (i.e., treat $t = 6$ ms as $t = 0$), this equation can be put in form

$$v_C(t) = V_0\, e^{-t/\tau} = 7.68\, e^{-t/\tau} \text{ V} \quad t \geq 0$$

where V_0 is the peak voltage of $v_C(t)$. We can solve for the time constant

$$\tau_{RC,\text{meas}} = \frac{-\Delta t}{\ln\left(v_C(\Delta t)/V_0\right)}$$

by choosing measured voltage data points $v_C(\Delta t)$ where $t = \Delta t$ is the time interval between the peak voltage V_0 and the measured voltage(s) $v_C(\Delta t)$.

To try to minimize measurement errors, we will solve for the time constant using several data points and take an average.

$$\tau_{RC,1} = \frac{-1\text{ms}}{\ln(v_c(1\text{ms})/V_0)} = \frac{-1e-3}{\ln(4.60/7.68)} = 1.951e-3\text{ s} \quad \Rightarrow \quad \underline{\tau_{RC,1} = 1.951\text{ ms}},$$

$$\tau_{RC,2} = \frac{-2\text{ms}}{\ln(v_c(2\text{ms})/V_0)} = \frac{-2e-3}{\ln(2.78/7.68)} = 1.968e-3\text{ s} \quad \Rightarrow \quad \underline{\tau_{RC,2} = 1.968\text{ ms}}, \text{ and}$$

$$\tau_{RC,3} = \frac{-3\text{ms}}{\ln(v_c(3\text{ms})/V_0)} = \frac{-3e-3}{\ln(1.66/7.68)} = 1.9585e-3\text{ s} \quad \Rightarrow \quad \underline{\tau_{RC,3} = 1.9585\text{ ms}}$$

$$\tau_{RC} = (1.951 + 1.968 + 1.9585)/3 \quad \Rightarrow \quad \underline{\boldsymbol{\tau_{RC} = 1.96\text{ ms}}}$$

6) Next, we can use the definition of the *RC* circuit time constant τ_{RC} and the known/measured resistor values to determine the unknown capacitance *C*.

$$\tau_{RC} = RC \quad \Rightarrow \quad C = \tau_{RC}/R$$

where *R* is the equivalent resistance 'seen' by the capacitor in Figure 1. Solving for the capacitance *C*, we get

$$C = \tau_{RC}/R = \tau_{RC}/(R_1 + R_S)$$
$$= 1.96\text{ ms}/(146.87 + 50)$$
$$= 0.00196/196.87$$

$$\Rightarrow \quad \underline{\boldsymbol{C = 0.000009956\text{ F} = 9.956\ \mu\text{F}}}$$

Apparently, our capacitor had a 10 μF nominal/rated value.

7) Last, we can now draw our actual *RC* circuit with the measured component values as shown below in Figure 2.

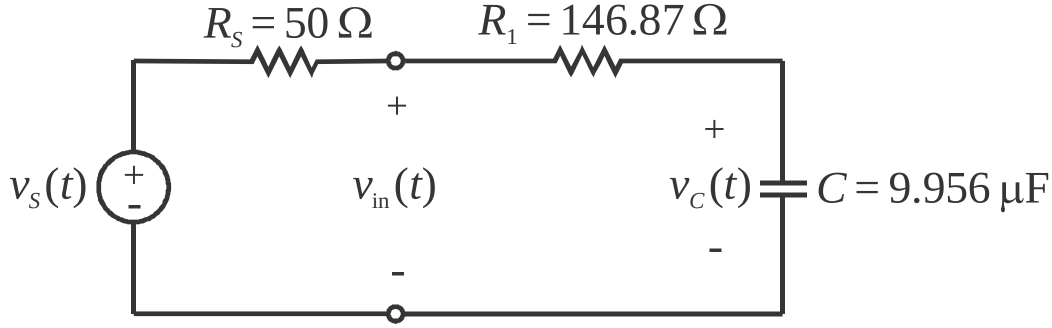


Figure 2 Measured first-order *RC* circuit