EE 220/220L Circuits I
Final Examination Review Topics

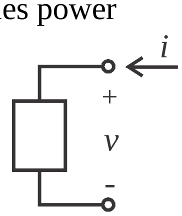
- This is NOT a complete list. Use as a starting point.

Chapter 1 Basic Concepts

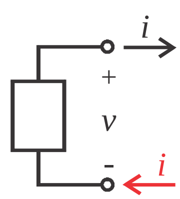
- Units & prefixes:

Prefix	Symbol	Multiplier	example
Tera-	T	10^{12}	$2\text{ THz} = 2 \times 10^{12}\text{ Hz}$
Giga-	G	10^{9}	$2.4\text{ GHz} = 2.4 \times 10^{9}\text{ Hz}$
Mega-	M	10^{6}	$101.9\text{ MHz} = 101.9 \times 10^{6}\text{ Hz}$
kilo-	k	10^{3}	$1340\text{ kHz} = 1340 \times 10^{3}\text{ Hz}$
centi-	c	10^{-2}	$10\text{ cm} = 10 \times 10^{-2}\text{ m}$
milli-	m	10^{-3}	$90\text{ mH} = 90 \times 10^{-3}\text{ mH}$
micro-	μ	10^{-6}	$47\ \mu\text{F} = 47 \times 10^{-6}\text{ F}$
nano-	n	10^{-9}	$100\text{ nH} = 100 \times 10^{-9}\text{ nH}$
pico-	p	10^{-12}	$150\text{ pF} = 150 \times 10^{-12}\text{ F}$

- Charge $q = \int i\, dt$ (C)
- Current $i = dq/dt$ (A)
- Voltage $v_{ab} = dw/dq$ (V)
- Work: $w = \int p\, dt$ (J)
- Power $p = dw/dt = dw/dq \bullet dq/dt = v\, i$ (W)
- Conservation of power : $\sum_{\text{circuit}} p = 0$
- Passive sign convention for power absorbed (see below)- $p > 0 \Rightarrow$ element absorbs power and if $p < 0 \Rightarrow$ element supplies power

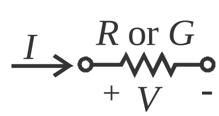


$p = v(i)$



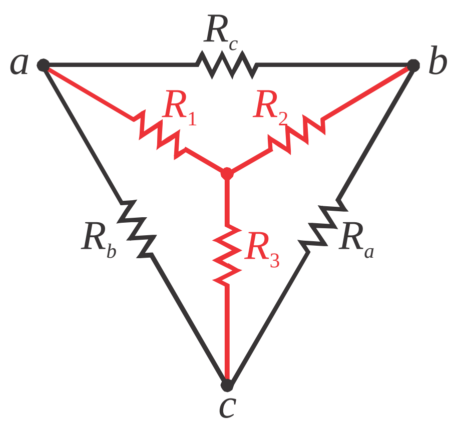
$p = -(v)(i)$

Chapter 2 Basic Laws



- Ohm's Law: $v = i\,R$ or $i = v\,G$
- Power & resistors: $p = v\,i = i^2 R = v^2/R = i^2/G = v^2 G$
- branches (b), nodes (n), & meshes/independent loops (l): $b = l + n - 1$
- Kirchoff's Current Law (KCL) : $\sum_{\text{node}} i_{\text{entering}} = 0$ or $\sum_{\text{node}} i_{\text{leaving}} = 0$ or $\sum_{\text{node}} i_{\text{entering}} = \sum_{\text{node}} i_{\text{leaving}}$
- Kirchoff's Voltage Law (KVL): $\sum_{\text{loop}} v_{\text{drops}} = 0$

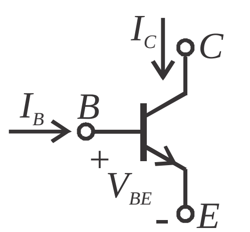
- Series resistors & voltage division: $R_{eq,\text{Series}} = \sum R_n$ & $v_n = v\dfrac{R_n}{R_{eq,S}}$
- Parallel resistors & current division: $R_{eq,\text{Parallel}} = \left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots \dfrac{1}{R_N}\right]^{-1}$ & $i_n = i\dfrac{R_{eq,P}}{R_n} = i\dfrac{G_n}{G_{eq,P}}$
- Wye ↔ Delta transformations



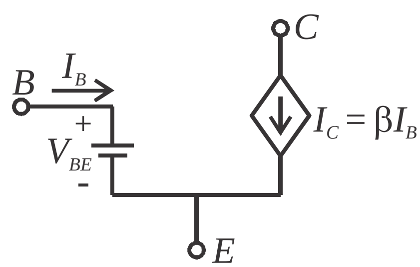
$\Delta \rightarrow \mathrm{Y}$	$\mathrm{Y} \rightarrow \Delta$
$R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c}$	$R_a = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$
$R_2 = \dfrac{R_a R_c}{R_a + R_b + R_c}$	$R_b = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$
$R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}$	$R_c = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$
Balanced $\Delta \rightarrow \mathrm{Y}$	Balanced $\mathrm{Y} \rightarrow \Delta$
When $R_a = R_b = R_c = R_\Delta$, $R_\mathrm{Y} = R_\Delta / 3$	When $R_1 = R_2 = R_3 = R_\mathrm{Y}$, $R_\Delta = 3R_\mathrm{Y}$

Chapter 3 Methods of Analysis

- Nodal Analysis
  1) Designate reference node and label all other node voltages (e.g., V_1, V_2, …).
  2) Apply KCL to all non-reference nodes, putting currents through resistors in terms of node voltages using Ohm's Law. Know how to deal with voltage sources (Case 1- $V_{\text{node}} = \pm V_S$ or Case 2- auxiliary & supernode equations). Express dependent sources in terms of node voltages.
  3) Solve resulting set of equations for node voltages.
- Mesh Analysis
  1) Select direction and label all mesh currents (e.g., I_1, I_2, …).
  2) Apply KVL around meshes, putting voltage drops across resistors in terms of mesh currents using Ohm's Law. Know how to deal with current sources (Case 1- $I_{\text{mesh}} = \pm I_S$ or Case 2- auxiliary & supermesh equations). Express dependent sources in terms of mesh currents.
  3) Solve resulting set of equations for mesh currents.
- DC npn bipolar junction transistor (BJT)



circuit symbol

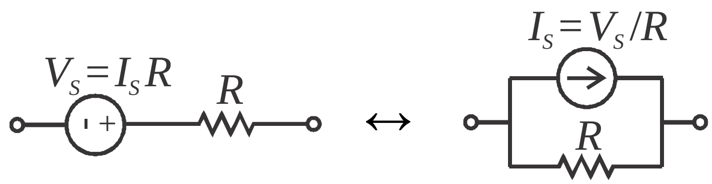


equivalent circuit model

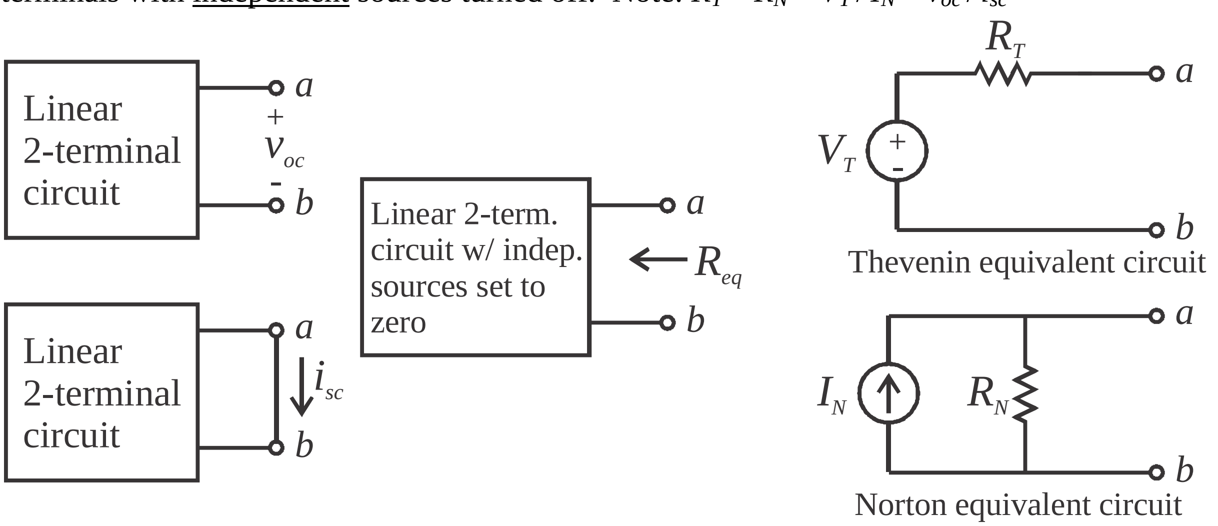
<u>Chapter 4 Circuit Theorems</u>

- Superposition Principle
  1) Set all <u>independent</u> sources to zero except one, i.e., $V_S = 0$ (short) and $I_S = 0$ (open). Leave dependent sources in circuit. Determine contributions to current(s) &/or voltage(s) of interest by this source using technique(s) of choice.
  2) Repeat step 1) for all remaining <u>independent</u> sources.
  3) Determine overall current(s) &/or voltage(s) by adding up contributions due to each of the individual independent sources.

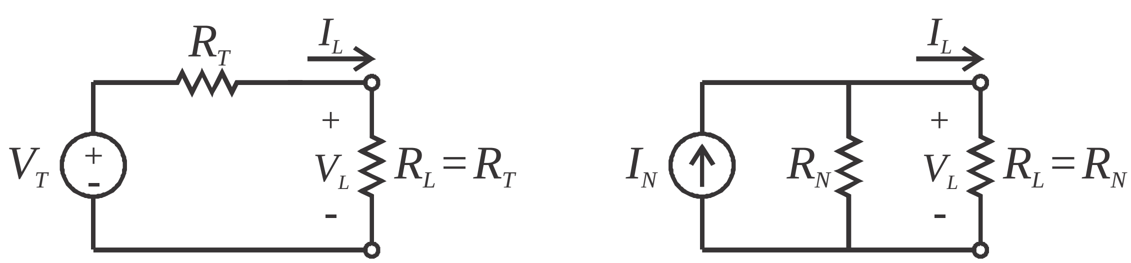
- Source transformation



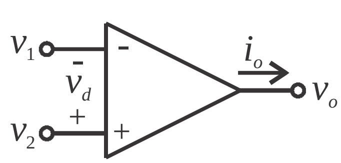
- Thevenin's and Norton's Theorems. A linear two-terminal circuit can be replaced by a Thevenin or Norton equivalent circuit. The Thevenin voltage V_T is equal to the open circuit voltage at the terminals. The Norton current I_N is equal to the short circuit current between the terminals. The Thevenin R_T and Norton R_N resistances are the equivalent resistances at the terminals with <u>independent</u> sources turned off. Note: $R_T = R_N = V_T / I_N = v_{oc} / i_{sc}$



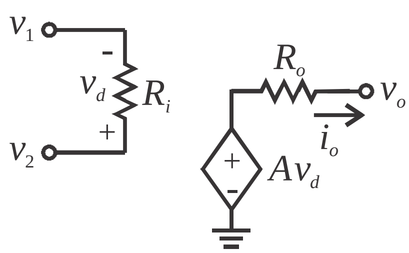
- Maximum power transfer to load occurs when $R_L = R_T$ (when using Thevenin equivalent circuit), or $R_L = R_N$ (when using Norton equivalent circuit)



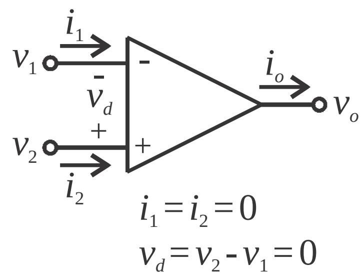
Chapter 5 Operational Amplifiers



circuit symbol



non-ideal equivalent circuit model

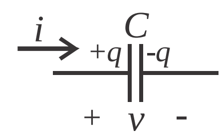


ideal circuit model

- Non-ideal op-amp equivalent circuit model where R_i is input resistance, R_o is output resistance, and A is open-loop voltage gain
- Know ideal op-amp circuit analysis both with single and multiple op-amps
- Know inverting, non-inverting, voltage follower, summing, difference, and instrumentation amplifier circuits.

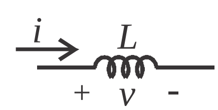
Chapter 6 Capacitors and Inductors

- Capacitor: $q = Cv$, $i = C\dfrac{dv}{dt}$, $v(t) = \dfrac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \dfrac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)$, $w = \dfrac{1}{2}Cv^2 = \dfrac{q^2}{2C}$
- Capacitor properties:



  1) Key parameters- capacitance C in Farads (F) & rated voltage
  2) Open circuit at DC, i.e., i =0, v = ?.
  3) Voltage cannot change abruptly, but current can change quickly.
  4) Ideal capacitors do not dissipate energy.
  5) Parallel capacitors $C_{\text{eq,parallel}} = C_1 + C_2 + \ldots + C_N$.
  6) Series capacitors $C_{\text{eq, series}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_N}\right]^{-1}$ & voltage division $v_n = v\left(\dfrac{C_{\text{eq, series}}}{C_n}\right)$.

- Inductor: $v = L\dfrac{di}{dt}$, $i(t) = \dfrac{1}{L}\int_{-\infty}^{t} v(t)\,dt = \dfrac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$, $w = \dfrac{1}{2}Li^2$
- Inductor properties:



  1) Key parameters- inductance L in Henries (H) & rated current
  2) Short circuit at DC, i.e., v =0, i = ?.
  3) Current cannot change abruptly.
  4) Voltage can change quickly.
  5) Ideal inductors do not dissipate energy.
  6) Series inductor s $L_{\text{eq,series}} = L_1 + L_2 + \ldots + L_N$.
  7) Parallel inductors $L_{\text{eq, parallel}} = \left[\dfrac{1}{L_1} + \dfrac{1}{L_2} + \ldots + \dfrac{1}{L_N}\right]^{-1}$ & current division $i_n = i\left(\dfrac{L_{\text{eq, parallel}}}{L_n}\right)$.

Chapter 7 First-Order Circuits

- Source-free RC circuit analysis: $v(t) = V_0 e^{-t/\tau} \quad t \geq 0$ where V_0 is the initial voltage across the capacitor and time constant τ = *RC*. *R* is equivalent resistance 'seen' by *C*.
- Source-free RL circuit analysis: $i(t) = I_0 e^{-t/\tau} \quad t \geq 0$ where I_0 is the initial current through the inductor and time constant τ = *L/R*. *R* is equivalent resistance 'seen' by *L*.
- Step response RC circuit analysis: $v(t) = \begin{cases} V_0 & t < 0 \\ V_{SS} + (V_0 - V_{SS})\, e^{-t/\tau} & t \geq 0 \end{cases}$ where $V_{SS} = v(\infty)$ is the steady-state voltage across the capacitor.
- Step response RL circuit analysis: $i(t) = \begin{cases} I_0 & t < 0 \\ I_{SS} + (I_0 - I_{SS})\, e^{-t/\tau} & t \geq 0 \end{cases}$ where $I_{SS} = i(\infty)$ is the steady-state current through the inductor.
- Singularity/switching functions. In particular, unit step function $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$.

Chapter 8 Second-Order Circuits

- Know how to determine initial & final conditions
- Source-free series *RLC* circuits: characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$
- Source-free parallel *RLC* circuits: characteristic equation $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$
- Three solution forms for currents/voltages depending on roots of characteristic equation:

$$x(t) = \begin{cases} A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{overdamped } (s_1 \,\&\, s_2 \text{ are real, negative, \& unequal}) \\ (A_2 + A_1 t)\, e^{-\alpha t} & \text{critically-damped } (s_1 = s_2 = -\alpha \text{ are real, negative, \& equal}) \\ e^{-\alpha t}\,(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)) & \text{underdamped } (s_{1/2} = -\alpha \pm j\omega_d \text{ are complex}) \end{cases}$$

- Step response parallel & series *RLC* circuits: current/voltage solution $x(t)$ consists of a forced response $x_f(t) = x_{SS} = x(\infty)$ (circuit at steady-state w/ source on) plus a natural response $x_n(t)$ (circuit w/ sources set to 0)

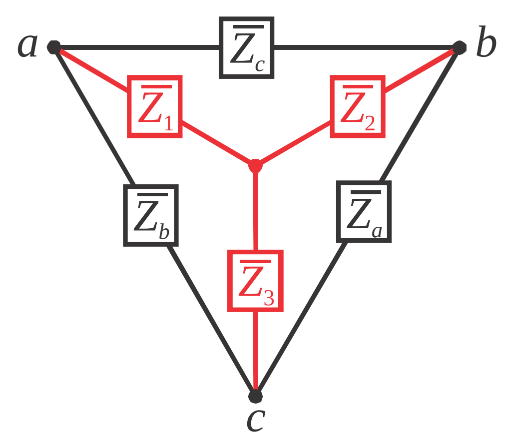
$$x(t) = x_f(t) + x_n(t) = \begin{cases} x_{SS} + A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{overdamped} \\ x_{SS} + (A_2 + A_1 t)\, e^{-\alpha t} & \text{critically-damped} \\ x_{SS} + e^{-\alpha t}\,(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)) & \text{underdamped} \end{cases}$$

- General second-order circuits (not included in final)

Chapter 9 Sinusoids and Phasors

- Sinusoids $x(t) = X_m \cos(\omega t + \theta) = X_m \sin(\omega t + \phi)$ - know what each of the components represents (e.g., $\omega = 2\pi f = 2\pi / T$) and how to convert from sin() form to cos() form
- Phasors $\bar{X} = X_m \angle \theta$ - complex number based on cos() form of sinusoid. Phasors allow for steady-state linear AC circuit analysis w/out need for difficult trigonometric identities.

- Phasor/frequency domain $\bar{X} = X_m \angle \theta$ $\leftrightarrow$ time-domain $x(t) = \mathrm{Re}\left\{\bar{X}\, e^{j\omega t}\right\} = X_m \cos(\omega t + \theta)$, using Euler's identity $e^{\pm jA} = \cos(A) \pm j\sin(A)$
- Impedance $\bar{Z}$ (Ω) for *R*, *L*, & *C*: $\bar{Z}_R = R$, $\bar{Z}_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$, & $\bar{Z}_L = j\omega L$
- Admittance $\bar{Y}$ (S) for *R*, *L*, & *C*: $\bar{Y}_R = \dfrac{1}{R} = G$, $\bar{Y}_C = j\omega C$, & $\bar{Y}_L = \dfrac{1}{j\omega L} = \dfrac{-j}{\omega L}$
- Phasor Ohm's Law $\bar{V} = \bar{I}\,\bar{Z}$ or $\bar{I} = \bar{V}\,\bar{Y}$
- Series equivalent impedances $\bar{Z}_{eq,S} = \bar{Z}_1 + \bar{Z}_2 \ldots + \bar{Z}_N$ & voltage division $\bar{V}_n = \bar{V}\,\dfrac{\bar{Z}_n}{\bar{Z}_{eq,S}}$
- Parallel equivalent impedances $\bar{Z}_{eq,P} = \left[\dfrac{1}{\bar{Z}_1} + \dfrac{1}{\bar{Z}_2} \ldots + \dfrac{1}{\bar{Z}_N}\right]^{-1}$ & current division $\bar{I}_n = \bar{I}\,\dfrac{\bar{Z}_{eq,P}}{\bar{Z}_n}$
- Wye↔Delta impedance transformations



$\Delta \rightarrow \mathrm{Y}$	$\mathrm{Y} \rightarrow \Delta$
$\bar{Z}_1 = \dfrac{\bar{Z}_b \bar{Z}_c}{\bar{Z}_a + \bar{Z}_b + \bar{Z}_c}$	$\bar{Z}_a = \dfrac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}{\bar{Z}_1}$
$\bar{Z}_2 = \dfrac{\bar{Z}_a \bar{Z}_c}{\bar{Z}_a + \bar{Z}_b + \bar{Z}_c}$	$\bar{Z}_b = \dfrac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}{\bar{Z}_2}$
$\bar{Z}_3 = \dfrac{\bar{Z}_a \bar{Z}_b}{\bar{Z}_a + \bar{Z}_b + \bar{Z}_c}$	$\bar{Z}_c = \dfrac{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}{\bar{Z}_3}$
Balanced $\Delta \rightarrow \mathrm{Y}$	Balanced $\mathrm{Y} \rightarrow \Delta$
When $\bar{Z}_a = \bar{Z}_b = \bar{Z}_c = \bar{Z}_\Delta$, $\bar{Z}_\mathrm{Y} = \bar{Z}_\Delta / 3$	When $\bar{Z}_1 = \bar{Z}_2 = \bar{Z}_3 = \bar{Z}_\mathrm{Y}$, $\bar{Z}_\Delta = 3\bar{Z}_\mathrm{Y}$

- Kirchoff's Current Law (KCL) in phasor domain: $\sum_{\text{node}} \bar{I}_{\text{in}} = 0$, $\sum_{\text{node}} \bar{I}_{\text{out}} = 0$, or $\sum_{\text{node}} \bar{I}_{\text{in}} = \sum_{\text{node}} \bar{I}_{\text{out}}$
- Kirchoff's Voltage Law (KVL) in phasor domain: $\sum_{\text{loop}} \bar{V}_{\text{drops}} = 0$

Chapter 10 Sinusoidal Steady-State Analysis

- Nodal Analysis w/ phasors- same process/procedure as for DC circuits (see Chapter 3)
- Mesh Analysis w/ phasors- same process/procedure as for DC circuits (see Chapter 3)
- Superposition Theorem - nearly same process/procedure as for DC circuits (see Chapter 4) with caveat that all results found using phasors must be converted back into time-domain before adding them up.
- Source Transformation w/ phasors- same process/procedure as for DC circuits (see Chapter 4)
- Thevenin and Norton Equivalent Circuits w/ phasors- same process/procedure as for DC circuits (see Chapter 4)

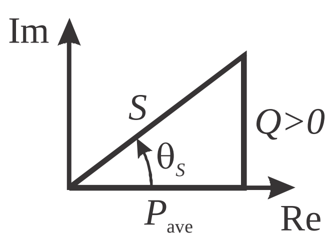
Chapter 11 AC Power Analysis

- Instantaneous Power $p(t) = v(t)\, i(t)$ (W)
- Effective/RMS values: $X_{\text{RMS}} \equiv \sqrt{\dfrac{1}{T}\displaystyle\int_{t_0}^{t_0+T} \left[x(t)\right]^2 dt}$. For a sinusoidal current or voltage $x(t) = X_m \cos(\omega t+\theta) \;\Rightarrow\; X_{\text{RMS}} = X_m / \sqrt{2}$
- Time-average real power $P_{\text{ave}} = \dfrac{1}{T}\displaystyle\int_0^T p(t)\, dt$ (W) in general. For sinusoidal steady-state circuits:

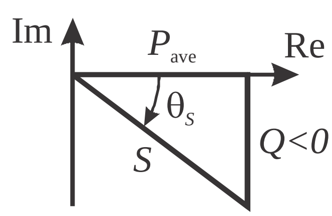
$$\begin{aligned} P_{\text{ave}} &= 0.5 V_m I_m \cos(\theta_V - \theta_I) = V_{rms} I_{rms} \cos(\theta_V - \theta_I) \\ &= 0.5\,\text{Re}[\bar{V}\,\bar{I}^*] = \text{Re}[\bar{V}_{rms}\bar{I}^*_{rms}] \\ &= 0.5\left|\bar{I}\right|^2 \text{Re}\ \bar{Z} = I^2_{rms}\,\text{Re}\ \bar{Z} \quad \text{(W)} \end{aligned}$$

- Maximum average power transfer occurs when $\underline{\bar{Z}_L = \bar{Z}_T^*}$ where $\bar{Z}_T$ is the Thevenin impedance of the source
- Apparent power $S = V_{\text{rms}}\, I_{\text{rms}} = 0.5\, V_{\text{m}}\, I_{\text{m}}$ (VA)
- Power factor $pf = P_{\text{ave}} / S = \cos(\theta_V - \theta_I) = \cos(\theta_z) = \cos(\theta_S)$ **leading** if $\theta_z < 0$ & **lagging** if $\theta_z > 0$
- Complex power $\equiv \bar{S} = S\angle\theta_s = P_{\text{ave}} + jQ = 0.5\bar{V}\,\bar{I}^* = \bar{V}_{rms}\bar{I}^*_{rms}$ (VA) where $P_{\text{ave}} \equiv$ time-average real power $= \text{Re}\ \bar{S}$ (W) and $Q \equiv$ reactive power $= \text{Im}\ \bar{S}$ (VAR)
- Power triangle

Resistive-inductive load (lagging *pf*)



Resistive-inductive load (leading *pf*)



- Conservation of AC power $\sum \bar{S}_{\text{supplied}} = \sum \bar{S}_{\text{absorbed}}$
- Power factor correction (not included in final)