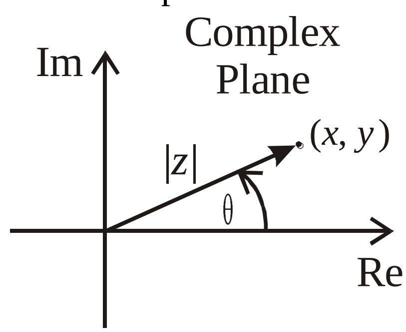
EE 220 Complex Number and Phasors Review

1) $j=\sqrt{-1}$ and $j^2=\left(\sqrt{-1}\right)^2=-1$

2) **Rectangular format**: The complex number z is defined as $z=x+jy$ where x is the real part and jy is the imaginary part.

3) **Exponential format**: The complex number z is defined as $z=|z|e^{j\theta}$ where $|z|=\sqrt{x^2+y^2}$ is the magnitude and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$ is the angle. This format is convenient for the multiplication and division operations for complex numbers.

4) **Polar or vector format:** The complex number z is defined as $z=|z|\angle\theta$. This format is convenient for the multiplication and division operations for complex numbers.



5) **Addition:** $z_1+z_2=(x_1+x_2)+j(y_1+y_2)$ the real parts are added to real parts and imaginary parts are added to imaginary parts.

6) **Subtraction:** $z_1-z_2=(x_1-x_2)+j(y_1-y_2)$ the real parts subtract from real parts and imaginary parts subtract from imaginary parts.

7) **Multiplication:** $z_1z_2=(x_1x_2-y_1y_2)+j(x_1y_2+x_2y_1)$ or $z_1z_2=|z_1||z_2|\angle(\theta_1+\theta_2)$. Note a real number multiplied by an imaginary number yields an imaginary number [e.g., $(x_1)(jy_2)=jx_1y_2$]. Further, a real number multiplied by a real number yields a real number and an imaginary number multiplied by an imaginary number also yields a real number [e.g., $(x_1)(x_2)=x_1x_2$ and $(jy_1)(jy_2)=j^2y_1y_2=-y_1y_2$].

8) **Complex conjugate:** $z^*=(x+jy)^*=x-jy$, $z^*=(|z|e^{j\theta})^*=|z|e^{-j\theta}$, or $z^*=(|z|\angle\theta)^*=|z|\angle-\theta$ (changes sign of imaginary part).

9) **Magnitude/Absolute value:** $|z|^2=zz^*=(x+jy)(x+jy)^*=x^2+y^2$ where $|z|=\sqrt{x^2+y^2}$ is the magnitude of z.

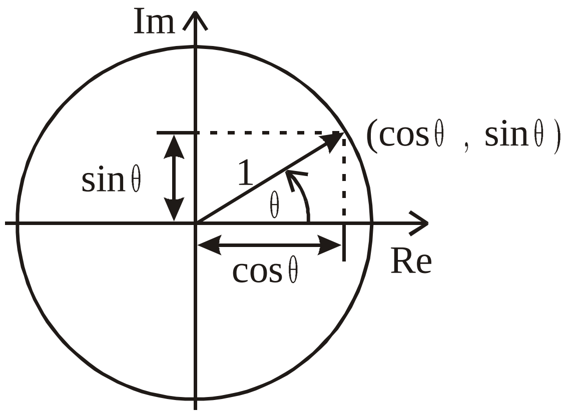
10) **Inverse:** $z^{-1}=\dfrac{1}{z}=\dfrac{x}{x^2+y^2}+j\dfrac{-y}{x^2+y^2}$

11) **Division:** $\frac{z_1}{z_2} = z_1 z_2^{-1} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + j\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2} = \frac{|z_1|\angle\theta_1}{|z_2|\angle\theta_2} = \frac{|z_1|}{|z_2|}\angle(\theta_1 - \theta_2)$.

12) The **Real operation:** $\mathrm{Re}(z) = x$ and **Imaginary operation:** $\mathrm{Im}(z) = y$ yield the magnitude of the real and imaginary parts of a complex number.

13) **Square root**: $\sqrt{z} = \sqrt{|z|}\,\angle\frac{\theta}{2} = \sqrt{|z|}\,\underline{|\frac{\theta}{2}}$

14) **Euler's Identity:** $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$ where $\left|e^{\pm j\theta}\right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$



15) **Phasors (AKA: frequency-domain):** complex numbers can be used to represent time-domain sinusoids. Here, the sinusoid $x(t) = A\cos(\omega t + \phi)$ is represented as $\bar{X} = Ae^{j\phi} = A\angle\phi$. The ωt time dependence is implied. Phasors for sinusoids **at the same frequency** can make adding, subtracting, multiplying, and dividing sinusoidal functions much easier; phasors are treated like any complex numbers.

16) To convert **phasors** back to the **time-domain** sinusoids they represent:
    a. multiply the phasor ($Ae^{j\phi}$ or $A\angle\phi$) by $e^{j\omega t}$ or $1\angle\omega t$,
    b. apply Euler's Identity, and
    c. find/keep the real part using the Re( ) operation.

e.g.,

$$\begin{aligned}\mathrm{Re}\{Ae^{j\phi}e^{j\omega t}\} &= \mathrm{Re}\{Ae^{j(\omega t+\phi)}\} \\ &= \mathrm{Re}\{A\cos(\omega t+\phi) + jA\sin(\omega t+\phi)\} \\ &= A\cos(\omega t+\phi)\end{aligned}$$

17) **Derivative**: $\frac{dv(t)}{dt} \Leftrightarrow j\omega\bar{V}$

18) **Integral**: $\int v(t)\,dt \Leftrightarrow \frac{\bar{V}}{j\omega}$