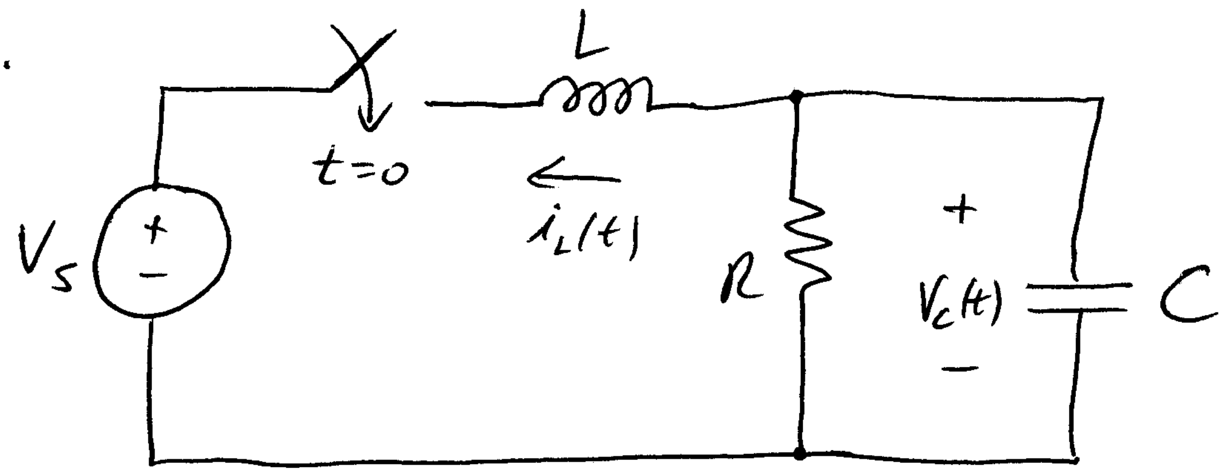
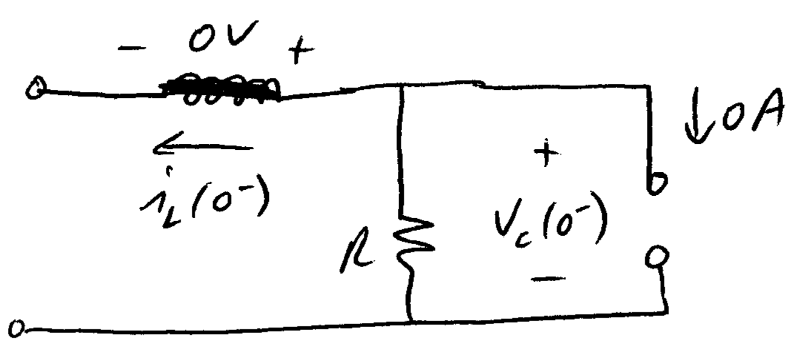
ex.



Find $i_L(t)$ & $V_c(t)$ when $V_s = 5V$, $L = 20mH$, $R = 1k\Omega$, and $C = 10nF$.

1) Find initial conditions

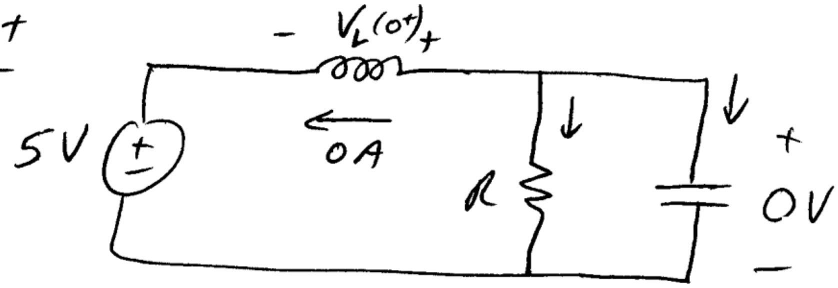
@ $t=0^-$ (Switch open & steady-state)



$$i_L(0^-) = 0 = i_L(0^+)$$

$$V_c(0^-) = 0 = V_c(0^+)$$

1) cont. $@\, t = 0^+$



Apply KCL $\quad 0A + \frac{0V}{R} + C\left.\frac{dV_c(t)}{dt}\right|_{t=0^+} = 0$

$$\hookrightarrow \left.\frac{dV_c}{dt}\right|_{t=0^+} = 0$$

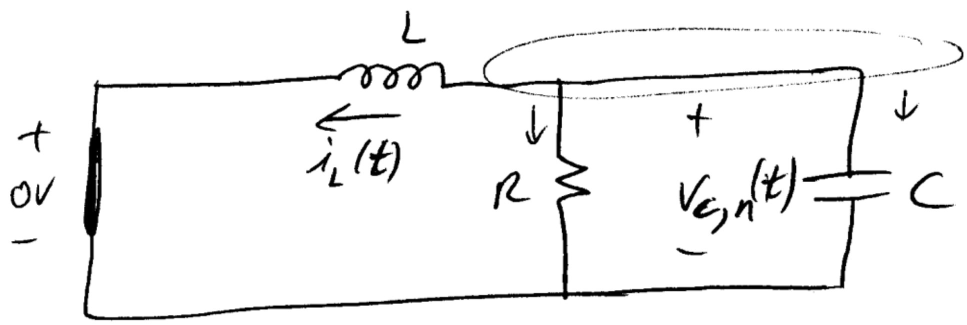
Apply KVL $\quad -5V - V_L(0^+) + 0 = 0$

$$V_L(0^+) = L\left.\frac{di_L}{dt}\right|_{t=0^+} = -5$$

$$\hookrightarrow \left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{-5}{20\times10^{-3}} = -250\ A/s$$

2) Find Natural Response $\quad t \geq 0$

→ set indep. sources to zero



Apply KCL $\quad i_L(t) + \frac{V_{c,n}(t)}{R} + C\frac{dV_{c,n}(t)}{dt} = 0$

$$\frac{1}{L}\int_{-\infty}^{t} V_{c,n}(t)\,dt + \frac{V_{c,n}(t)}{R} + C\frac{dV_{c,n}(t)}{dt} = 0$$

ex. cont.

2) cont taking $\frac{d}{dt}$

$$\frac{V_{c,n}}{L} + \frac{1}{R}\frac{dv_{c,n}}{dt} + \frac{1}{C}\frac{d^2V_{c,n}}{dt^2} = 0$$

$$\frac{d^2V_{c,n}}{dt^2} + \frac{1}{RC}\frac{dV_{c,n}}{dt} + \frac{1}{LC}V_{c,n} = 0$$

Assuming $V_{c,n} = A e^{st}$

Characteristic Eqn $\quad s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

Plug in R, L, + C values

$$s^2 + 100s + 5\times10^6 = 0$$

TI-68 root solver

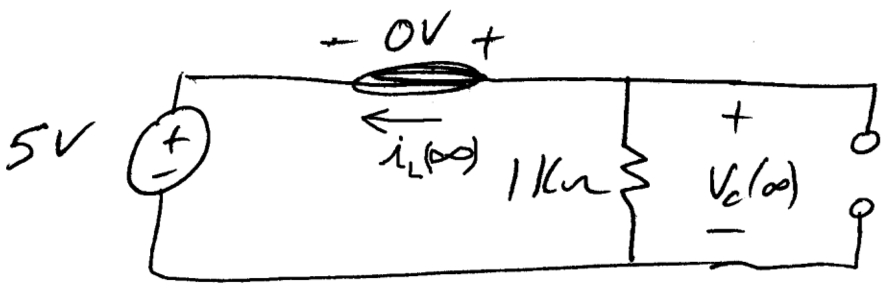
$$\left.\begin{aligned} s_1 &= -50 + j\,2235.5 \ s^{-1} \\ s_2 &= \underset{\alpha}{-50} - j\,\underset{\omega d}{2235.5} \ s^{-1} \end{aligned}\right\}$$ Underdamped

$$V_{c,n}(t) = e^{-50t}\left[A_1\cos(2235.5\,t) + A_2\sin(2235.5\,t)\right]$$

ex. cont.

3) Find Forced Response

let $t \to \infty$ (steady-state)



$$\Rightarrow V_c(\infty) = 5V = V_{c,f} \qquad i_L(\infty) = \frac{0-5}{1000} = -5mA$$

4) Find Total Response

$$V_c(t) = V_{c,N}(t) + V_{c,f}$$

$$V_c(t) = e^{-50t}\left[A_1 \cos(2235.5t) + A_2 \sin(2235.5t)\right] + 5$$

Apply $V_c(0) = 0 = (1)\left[A_1(1) + A_2(0)\right] + 5$

$$\hookrightarrow \underline{A_1 = -5V}$$

$$\frac{dV_c(t)}{dt} = e^{-50t}(-50)\left[-5\cos(2235.5t) + A_2 \sin(2235.5t)\right] + e^{-50t}\left[(-5)(-2235.5 \sin(2235.5t)) + A_2(2235.5\cos(2235.5t))\right]$$

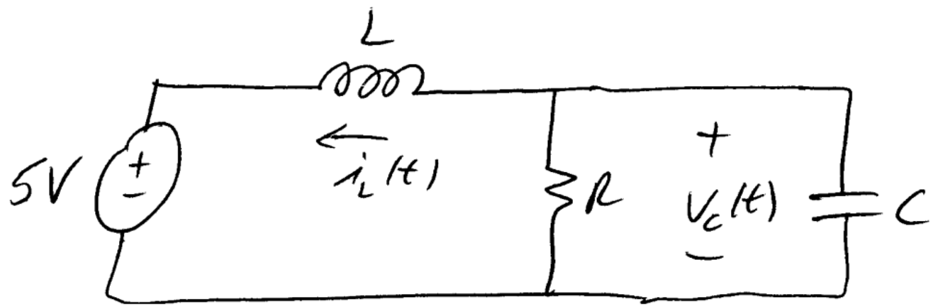
$$\left.\frac{dV_c}{dt}\right|_{t=0} = 0 = (1)(-50)\left[-5 + 0\right] + (1)\left[0 + A_2(2235.5)\right]$$

ex. cont.

4) cont. $A_2 = \frac{-(-50)(-5)}{2235.5} = -0.11183$

$$\boxed{V_c(t) = e^{-50t}\left[-5\cos(2235.5t) - 0.11183\sin(2235.5t)\right] + 5\text{ V} \quad t \geq 0}$$

Find $i_L(t)$



$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t}(V_c(t) - 5)\,dt$$

OR using KCL

$$i_L(t) = \frac{0 - V_c(t)}{R} + C\,\frac{d(0 - V_c(t))}{dt}$$

OR

$$i_L(t) = e^{-50t}\left[B_1\cos(2235.5t) + B_2\sin(2235.5t)\right] + i_L(\infty)$$

$$\boxed{i_L(t) = e^{-50t}\left[5\cos(2235.5t) - 111.72\sin(2235.5t)\right] - 5\text{ mA} \quad t \geq 0}$$

Step Response Parallel RLC circuit (underdamped)

$n := 0..2000 \qquad t_n := \frac{n}{33000} \quad \text{s}$

$vC(x) := e^{-50 \cdot x} \cdot (-5 \cdot \cos(2235.5 \cdot x) - 0.11183 \cdot \sin(2235.5 \cdot x)) + 5 \quad \text{V}$

$iL(x) := e^{-50 \cdot x} \cdot (5 \cdot \cos(2235.5 \cdot x) - 117.19 \cdot \sin(2235.5 \cdot x)) - 5 \quad \text{mA}$