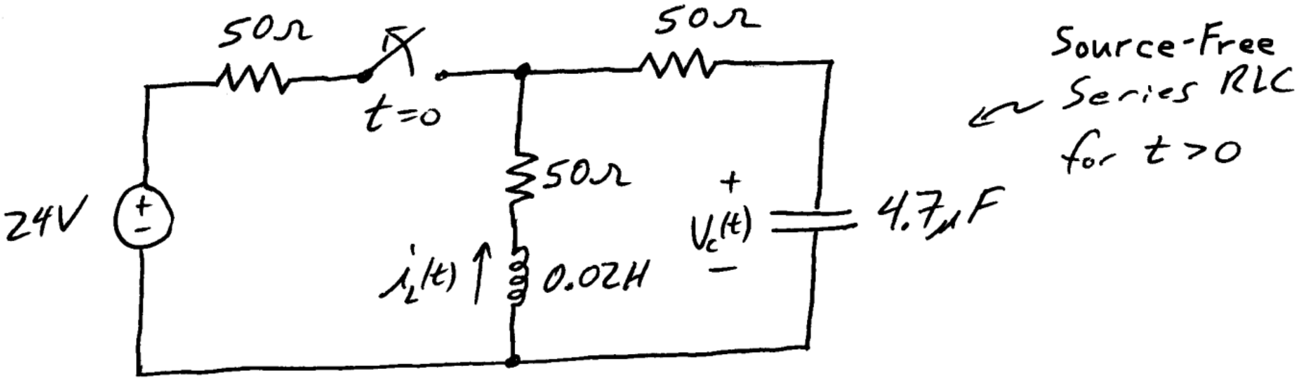
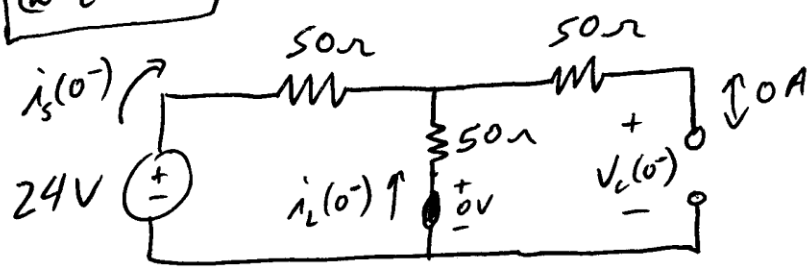
ex. Find $i_L(t)$ and $V_c(t)$ for $t>0$ for the circuit shown.



Step 1 Determine initial conditions

$\boxed{@\ t=0^-}$ (CKt @ steady-state, $i_c = 0$ + $V_L = 0$)



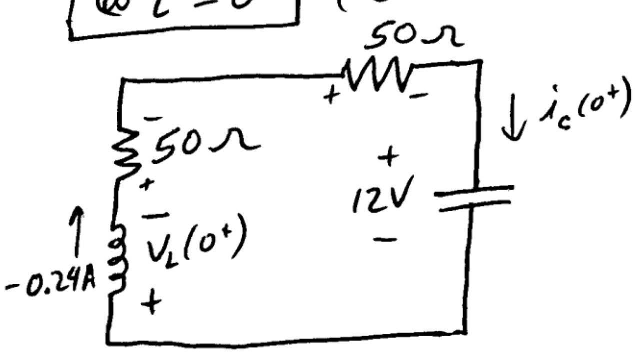
By Ohm's Law, $i_s(0^-) = \dfrac{24V}{50+50\Omega} = 0.24A$

$$i_L(0^-) = -i_s(0^-) = \underline{\underline{-0.24A = i_L(0)}}$$

By KVL on righthand loop $\quad 0 + i_L(0^-)50 + 0(50) + V_c(0^-) = 0$

$$V_c(0^-) = +0.24(50) = \underline{\underline{12V = V_c(0)}}$$

$\boxed{@\ t=0^+}$ (CKt in transition)



$$i_c(0^+) = -0.24A = C\left.\frac{dV_c}{dt}\right|_{t=0^+}$$

$$\left.\frac{dV_c}{dt}\right|_{t=0^+} = \frac{-0.24A}{4.7\times10^{-6}F}$$

$$\underline{\underline{\left.\frac{dV_c}{dt}\right|_{t=0^+} = -51{,}064\ V/s}}$$

Apply KVL around the loop

$$V_L(0^+) + (-0.24)(50+50) + 12 = 0$$

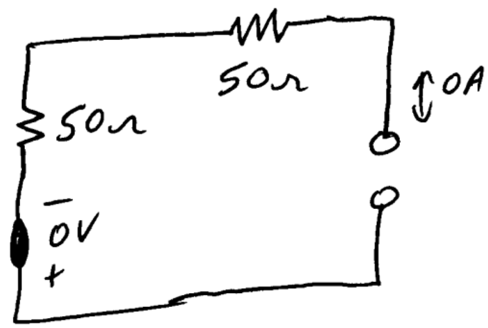
$$V_L(0^+) = 0.02 \left.\frac{di_L}{dt}\right|_{t=0^+} = 12$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = 600 \text{ A/s}$$

Step 2 Find final conditions (steady-state)

@ $t \to \infty$

No current & No voltages ⇒



$i_L(\infty) = 0$

$V_C(\infty) = 0$

Step 3 Differential & Characteristic Eqns

$$(8.4) \quad \frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0$$

&

$$(8.8) \quad s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + \left(\frac{50+50}{0.02}\right)s + \frac{1}{0.02(4.7\times10^{-6})} = 0$$

$$s^2 + 5000\,s + 10{,}638{,}298 = 0$$

Find roots using TI-68

$$s_1 = -2500 + j\,2094.8 \ \text{s}^{-1}$$

$$s_2 = -2500 - j\,2094.8 \ \text{s}^{-1}$$

Underdamped!

From s_1 & s_2 and/or eq'ns:

$$\alpha = |\mathcal{R}e(s_{1,2})| = \frac{R}{2L} = 2500 \text{ Np/s} \qquad (8.11)$$

damped → $\omega_d = |Im(s_{1,2})| = \sqrt{\omega_0^2 - \alpha^2} = 2094.8 \text{ rad/s}$ (8.22 a,b)

undamped → $\omega_0 = \frac{1}{\sqrt{LC}} = 3261.6 \text{ rad/s}$ (8.11)

<u>Step 4 General Solution</u>

(8.26) $i_L(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$

<u>Step 5 Find particular sol'n using initial conditions</u>

$$i_L(0) = -0.24 = e^0\left[B_1 \cos(0) + B_2 \underbrace{\sin(0)}_{\to 0}\right]$$

$\hookrightarrow \underline{B_1 = -0.24}$

$$i_L(t) = e^{-\alpha t}\left[-0.24 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$\frac{di_L}{dt} = -\alpha e^{-\alpha t}\left[-0.24 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right] + e^{-\alpha t}\left[+0.24\, \omega_d \sin(\omega_d t) + B_2 \omega_d \cos(\omega_d t)\right]$$

$$\left.\frac{di_L}{dt}\right|_{t=0} = 600 = -2500 e^0\left[-0.24 \overbrace{\cos(0)}^{1} + B_2 \underbrace{\sin(0)}_{\to 0}\right] + e^0\left[0.24(2094.8)\underbrace{\sin(0)}_{\to 0} + B_2 (2094.8) \overbrace{\cos(0)}^{1}\right]$$

$$600 = 600 + B_2(2094.8)$$

$\hookrightarrow \underline{B_2 = 0}$

Step 5 cont.

$$i_L(t) = -0.24\, e^{-2500t} \cos(2094.8t) \quad (A) \quad t \geq 0$$

Step 6 Find $V_c(t)$

Note that $i_L(t) = i_c(t)$. Therefore

$$V_c(t) = 1/C \int_{t=0}^{t} i_L(t)\, dt + V_c(0)$$

$$= \frac{1}{4.7\times10^{-6}} \int_{t=0}^{t} -0.24\, e^{-2500t} \cos(2094.8t)\, dt + 12\ V$$

use $\displaystyle\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}\left[a\cos(bx) + b\sin(bx)\right]}{a^2+b^2}$

$$V_c(t) = (-51{,}063.83)\left.\frac{e^{-2500t}\left[-2500\cos(2094.8t) + 2094.8\sin(2094.8t)\right]}{(-2500)^2 + (2094.8)^2}\right|_{t=0}^{t}$$

$$+12\ V$$

$$= -0.0048\Big[e^{-2500t}\big(-2500\cos(2094.8t) + 2094.8\sin(2094.8t)\big) - e^{0}\big(-2500\cos(0) + 2094.8\sin(0)\big)\Big]$$

$$+12$$

$$V_c(t) = e^{-2500t}\left[12\cos(2094.8t) - 10\sin(2094.8t)\right] (V) \quad t \geq 0$$

Source Free Series RLC circuit (underdamped)

$n := 0..100 \qquad t_n := \frac{0.003}{100} n \quad s$

$iL_n := -0.24 \cdot e^{-2500 \cdot t_n} \cdot \cos(2094.8 \cdot t_n) \qquad A,\ t > 0$

$vc_n := e^{-2500 \cdot t_n} \cdot (12 \cos(2094.8 \cdot t_n) - 10 \cdot \sin(2094.8 \cdot t_n)) \qquad V,\ t > 0$