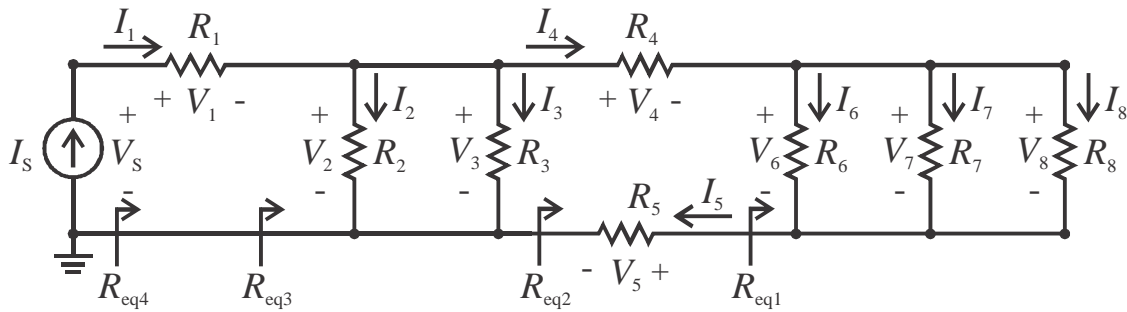


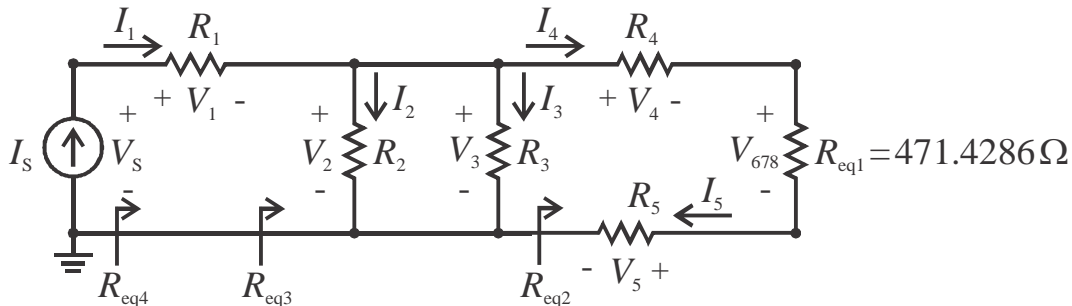
Example- Use circuit reduction techniques to solve for indicated variables



given that $I_s = 20 \text{ mA}$, $R_1 = 100 \Omega$, $R_2 = 500 \Omega$, $R_3 = 600 \Omega$, $R_4 = 200 \Omega$, $R_5 = 300 \Omega$, $R_6 = 1 \text{ k}\Omega$, $R_7 = 1.5 \text{ k}\Omega$, and $R_8 = 2.2 \text{ k}\Omega$.

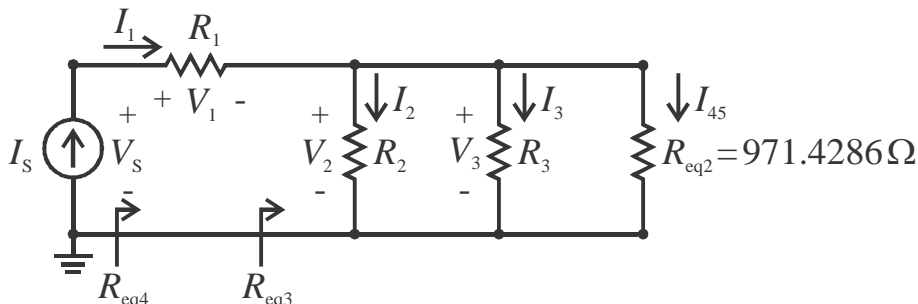
Step 1

Note resistors R_6 , R_7 , & R_8 are in parallel. Combine & replace with equivalent resistance $R_{eq1} = R_6 // R_7 // R_8 = [1000^{-1} + 1500^{-1} + 2200^{-1}]^{-1} = 471.4286 \Omega$ and designate the common parallel voltage $V_{678} = V_6 = V_7 = V_8$. Note that currents I_6 , I_7 , and I_8 are ‘lost’ in the new equivalent circuit.



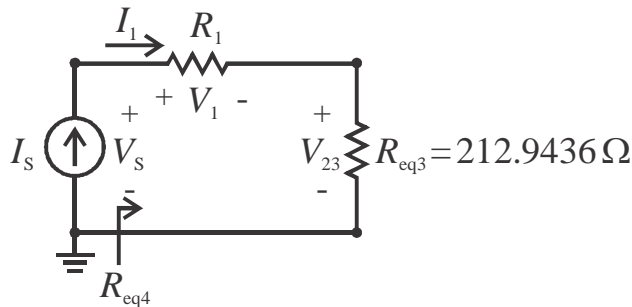
Step 2

Note resistors R_4 , R_{eq1} , & R_5 are in series. Combine & replace with equivalent resistance $R_{eq2} = R_4 + R_{eq1} + R_5 = 200 + 471.4286 + 300 = 971.4286 \Omega$ and designate the common series current $I_{45} = I_4 = I_5$. Note that voltages V_4 , V_{678} , and V_5 are ‘lost’ in the new equivalent circuit.



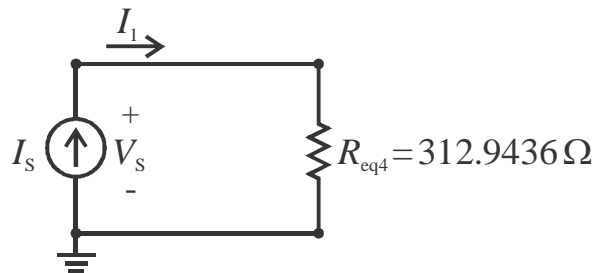
Step 3

Note resistors R_2 , R_3 , & R_{eq2} are in parallel. Combine & replace with equivalent resistance $R_{eq3} = R_2 // R_3 // R_{eq2} = [500^{-1} + 600^{-1} + 971.4286^{-1}]^{-1} = \underline{212.9436\ \Omega}$ and designate the common parallel voltage $V_{23} = V_2 = V_3$. Note that currents I_2 , I_3 , and I_{45} are 'lost' in the new equivalent circuit.



Step 4

Note that resistors R_1 and R_{eq3} are in series. Combine and replace with equivalent resistance $R_{eq4} = R_1 + R_{eq3} = 100 + 212.9436 = \underline{312.9436\ \Omega}$ with the common series current I_1 . Note that voltages V_1 and V_{23} are 'lost' in the new equivalent circuit.



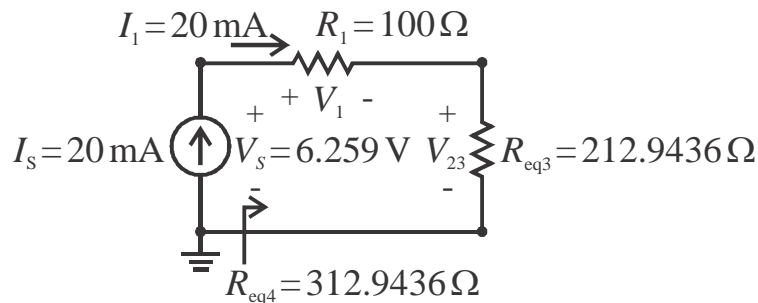
Now, we can start determining the unknown current I_1 and voltage V_s for this circuit. Then, we will 're-expand' or undo our circuit reduction to find the remaining currents and voltages.

By KCL, $I_1 = I_s = 20\ \text{mA}$

By Ohm's Law, $V_s = I_1 R_{eq4} = 20 \times 10^{-3} (312.9436) \rightarrow V_s = 6.2589\ \text{V}$

Step 5

Knowing I_1 and V_S , go back to the circuit of step 3 and find V_1 and V_{23} .



There are multiple ways to calculate V_1 and V_{23} :

By Ohm's Law, $V_1 = I_1 R_1 = 20 \cdot 10^{-3} (100) = 2 \text{ V}$,

OR

by voltage division, $V_1 = V_S (R_1 / R_{eq4}) = 6.25887 (100 / 312.9) = 2 \text{ V}$.

Next, by Ohm's Law, $V_{23} = I_1 R_{eq3} = 20 \cdot 10^{-3} (212.9436) = 4.25887 \text{ V}$,

OR

by KVL, $-V_S + V_1 + V_{23} = -6.25887 + 2 + V_{23} = 0 \rightarrow V_{23} = 4.25887 \text{ V}$,

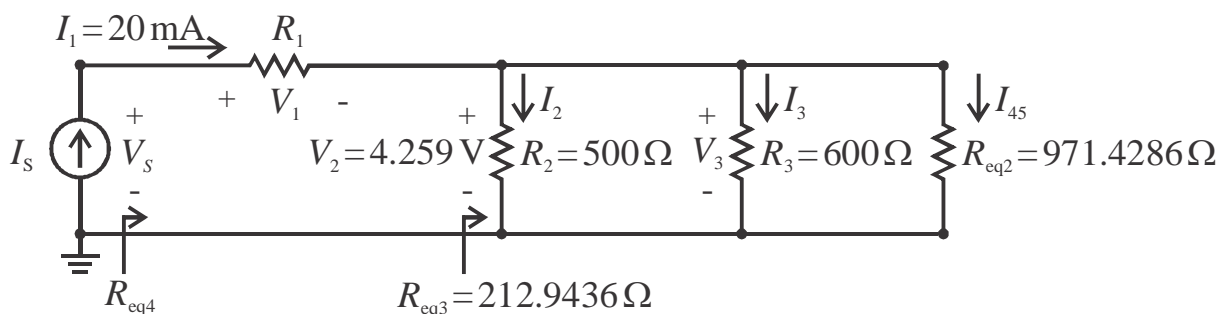
OR

by voltage division, $V_{23} = V_S (R_{eq3} / R_{eq4}) = 6.25887 (212.9 / 312.9) = 4.25887 \text{ V}$

So, $V_1 = 2 \text{ V}$ and $V_{23} = V_2 = V_3 = 4.2589 \text{ V}$.

Step 6

Knowing I_1 , V_1 , V_2 , V_3 , & V_S , go back to circuit of step 2 and find I_2 , I_3 , & I_{45} .



Again, there are multiple ways to calculate I_2 , I_3 , & I_{45} :

By Ohm's Law, $I_2 = V_2 / R_2 = 4.259 / 500 = 8.51774 \text{ mA}$,

OR

by current division, $I_2 = I_1 (R_{eq3} / R_2) = 20(212.944 / 500) = 8.51774 \text{ mA}$.

Next, by Ohm's Law, $I_3 = V_3 / R_3 = 4.259 / 600 = 7.09812 \text{ mA}$,

OR

by current division, $I_3 = I_1 (R_{eq3} / R_3) = 20(212.944 / 600) = 7.09812 \text{ mA}$.

Last, by Ohm's Law, $I_{45} = V_3 / R_{eq2} = 4.259 / 971.4286 = 4.38413 \text{ mA}$,

OR

by current division, $I_{45} = I_1 (R_{eq3} / R_{eq2}) = 20(212.94 / 971.43) = 4.3841 \text{ mA}$,

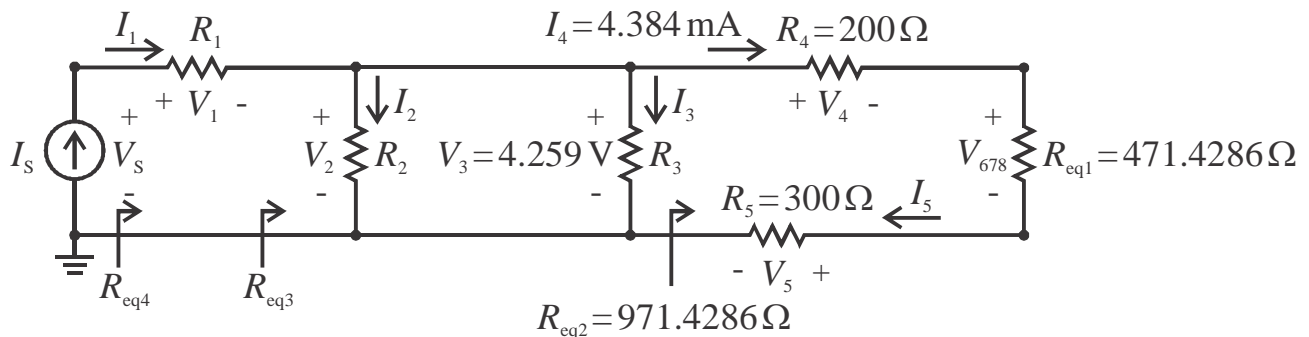
OR

by KCL, $I_1 = I_2 + I_3 + I_{45} \rightarrow I_{45} = 20 - 8.5177 - 7.0981 = 4.3841 \text{ mA}$.

So, $I_2 = 8.5177 \text{ mA}$, $I_3 = 7.0981 \text{ mA}$, and $I_{45} = I_4 = I_5 = 4.3841 \text{ mA}$.

Step 7

Knowing $I_1, I_2, I_3, I_4, I_5, V_1, V_2, V_3$, & V_S , go back to circuit of step 1 and find V_4, V_5 , & V_{678} .



Again, there are multiple ways to calculate V_4, V_5 , & V_{678} :

By Ohm's Law, $V_4 = I_4 R_4 = 4.38414 * 10^{-3} (200) = 0.87683 \text{ V}$,

OR

by voltage division, $V_4 = V_3 (R_4 / R_{eq2}) = 4.25887 (200 / 971.4286) = 0.87683 \text{ V}$.

Next, by Ohm's Law, $V_5 = I_5 R_5 = 4.38414 * 10^{-3} (300) = 1.31524 \text{ V}$,

OR

by voltage division, $V_5 = V_3 (R_5 / R_{eq2}) = 4.25887 (300 / 971.4286) = 1.31524 \text{ V}$.

Last, by Ohm's Law, $V_{678} = I_4 R_{eq1} = 4.38414 \times 10^{-3} (471.4286) = 2.06681 \text{ V}$,

OR

by voltage division, $V_{678} = V_3 (R_{eq1} / R_{eq2}) = 4.25887 (471.4286 / 971.4286) = 2.066805 \text{ V}$,

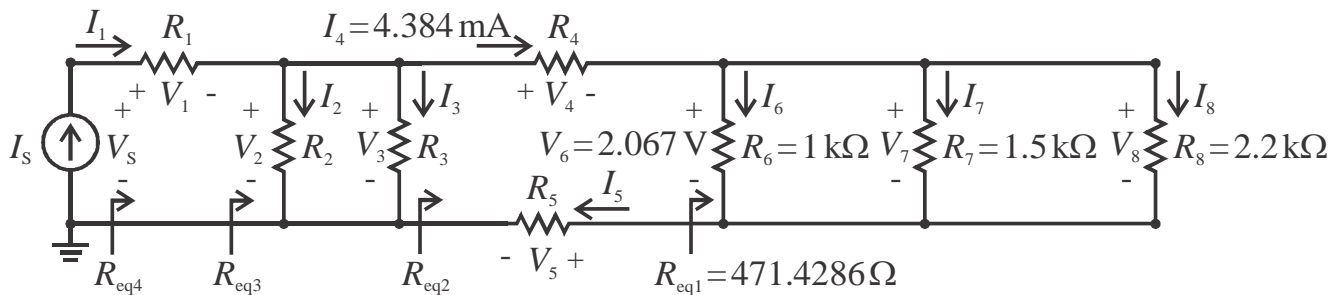
OR

by KVL, $V_3 = V_4 + V_{678} + V_5 \rightarrow V_{678} = 4.25887 - 0.87683 - 1.31524 = 2.0668 \text{ V}$.

So, $V_4 = 0.8768 \text{ V}$, $V_5 = 1.3152 \text{ V}$, and $V_{678} = V_6 = V_7 = V_8 = 2.0668 \text{ V}$.

Step 8

Knowing $I_1, I_2, I_3, I_4, I_5, V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$, & V_S , go back to original circuit and find I_6, I_7 , & I_8 .



Again, there are multiple ways to calculate I_6, I_7 , & I_8 :

By Ohm's Law, $I_6 = V_6 / R_6 = 2.06681 / 1000 = 2.06681 \text{ mA}$,

OR

by current division, $I_6 = I_4 (R_{eq1} / R_6) = 4.38414 (471.4286 / 1000) = 2.06681 \text{ mA}$.

Next, by Ohm's Law, $I_7 = V_7 / R_7 = 2.06681 / 1500 = 1.37787 \text{ mA}$,

OR

by current division, $I_7 = I_4 (R_{eq1} / R_7) = 4.38414 (471.4286 / 1500) = 1.37787 \text{ mA}$.

Last, by Ohm's Law, $I_8 = V_8 / R_8 = 2.06681 / 2200 = 0.93946 \text{ mA}$,

OR

by current division, $I_8 = I_4 (R_{eq1} / R_8) = 4.38414 (471.4286 / 2200) = 0.93946 \text{ mA}$,

OR

by KCL, $I_4 = I_6 + I_7 + I_8 \rightarrow I_8 = 4.38414 - 2.06681 - 1.37787 = 0.93946 \text{ mA}$.

So, $I_6 = 2.0668 \text{ mA}$, $I_7 = 1.3779 \text{ mA}$, and $I_8 = 0.9395 \text{ mA}$.