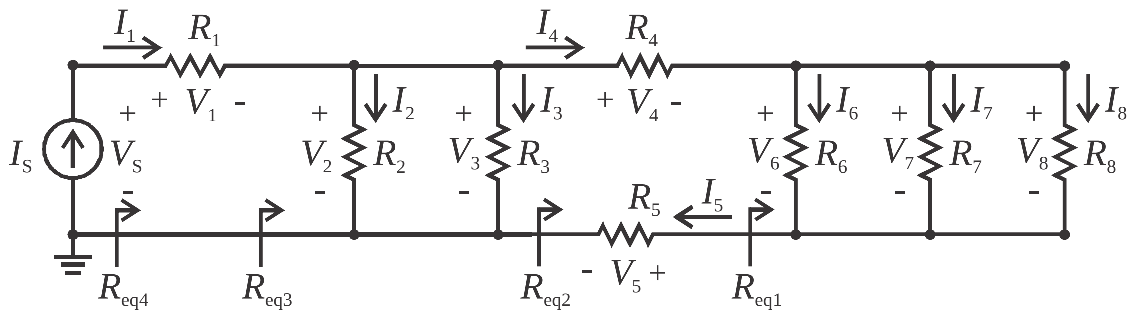
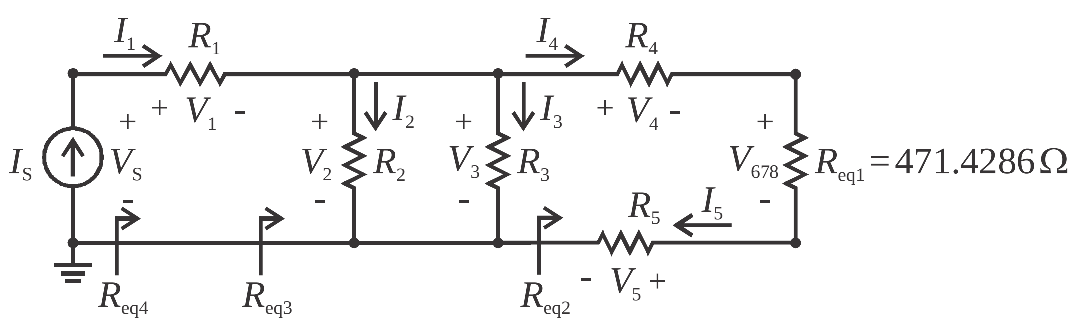
Example- Use circuit reduction techniques to solve for indicated variables



given that I_S = 20 mA, R_1 = 100 Ω, R_2 = 500 Ω, R_3 = 600 Ω, R_4 = 200 Ω, R_5 = 300 Ω, R_6 = 1 kΩ, R_7 = 1.5 kΩ, and R_8 = 2.2 kΩ.

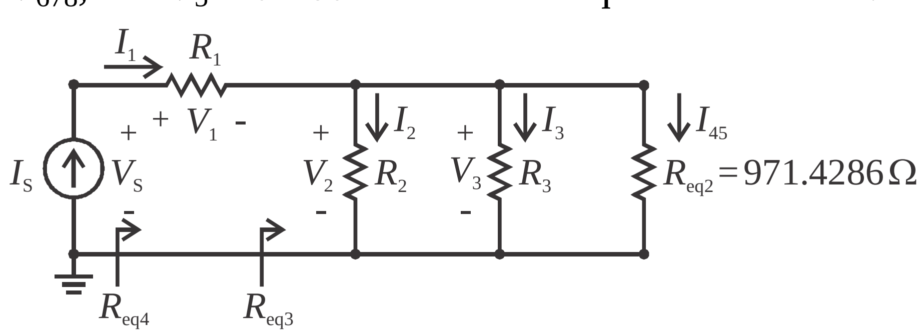
Step 1

Note resistors R_6, R_7, & R_8 are in parallel. Combine & replace with equivalent resistance $R_{eq1} = R_6 \,//\, R_7 \,//\, R_8 = [1000^{-1} + 1500^{-1} + 2200^{-1}]^{-1}$ = 471.4286 Ω and designate the common parallel voltage $V_{678} = V_6 = V_7 = V_8$. Note that currents I_6, I_7, and I_8 are 'lost' in the new equivalent circuit.



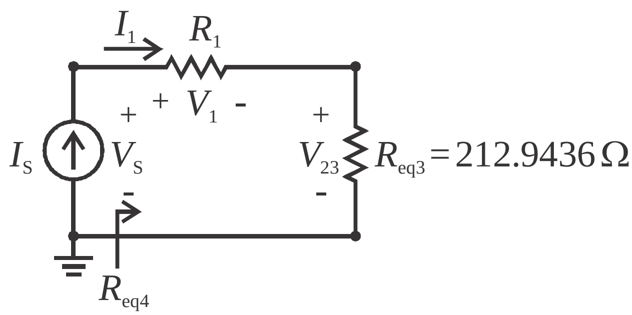
Step 2

Note resistors R_4, R_{eq1}, & R_5 are in series. Combine & replace with equivalent resistance $R_{eq2} = R_4 + R_{eq1} + R_5 = 200 + 471.4286 + 300$ = 971.4286 Ω and designate the common series current $I_{45} = I_4 = I_5$. Note that voltages V_4, V_{678}, and V_5 are 'lost' in the new equivalent circuit.



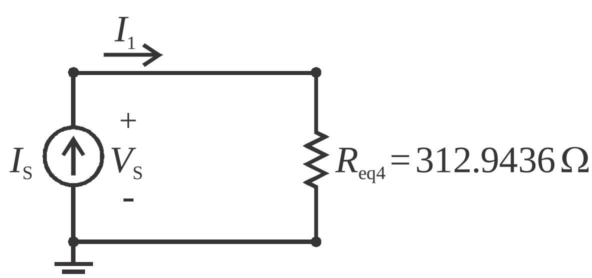
Step 3

Note resistors R_2, R_3, & R_{eq2} are in parallel. Combine & replace with equivalent resistance $R_{eq3} = R_2 \,//\, R_3 \,//\, R_{eq2} = [500^{-1} + 600^{-1} + 971.4286^{-1}]^{-1} = \underline{212.9436\,\Omega}$ and designate the common parallel voltage $V_{23} = V_2 = V_3$. Note that currents I_2, I_3, and I_{45} are 'lost' in the new equivalent circuit.



Step 4

Note that resistors R_1 and R_{eq3} are in series. Combine and replace with equivalent resistance $R_{eq4} = R_1 + R_{eq3} = 100 + 212.9436 = \underline{312.9436\,\Omega}$ with the common series current I_1. Note that voltages V_1 and V_{23} are 'lost' in the new equivalent circuit.



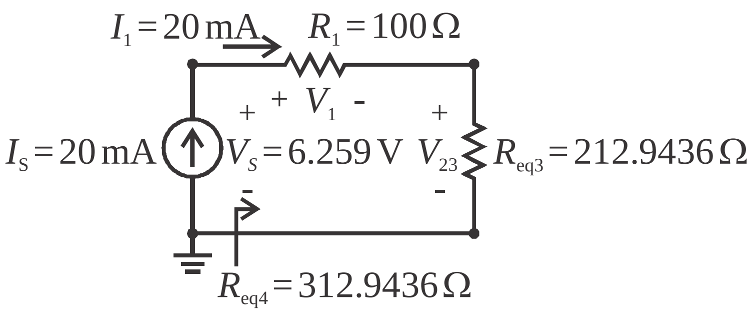
Now, we can start determining the unknown current I_1 and voltage V_S for this circuit. Then, we will 're-expand' or undo or circuit reduction to find the remaining currents and voltages.

By KCL, $\boxed{\boldsymbol{I_1 = I_S = 20\text{ mA}}}$

By Ohm's Law, $V_S = I_1\, R_{eq4} = 20{*}10^{-3}(312.9436) \rightarrow$ $\boxed{\boldsymbol{V_S = 6.2589\text{ V}}}$

Step 5

Knowing I_1 and V_S, go back to the circuit of step 3 and find V_1 and V_{23}.



There are multiple ways to calculate V_1 and V_{23}:

By Ohm's Law, $V_1 = I_1\, R_1 = 20{*}10^{-3}(100) = 2$ V,

OR

by voltage division, $V_1 = V_s\,(R_1/\,R_{eq4}) = 6.25887(100/312.9) = 2$ V.

Next, by Ohm's Law, $V_{23} = I_1\, R_{eq3} = 20{*}10^{-3}(212.9436) = 4.25887$ V,

OR

by KVL, $-V_s + V_1 + V_{23} = -6.25887 + 2 + V_{23} = 0 \rightarrow V_{23} = 4.25887$ V,

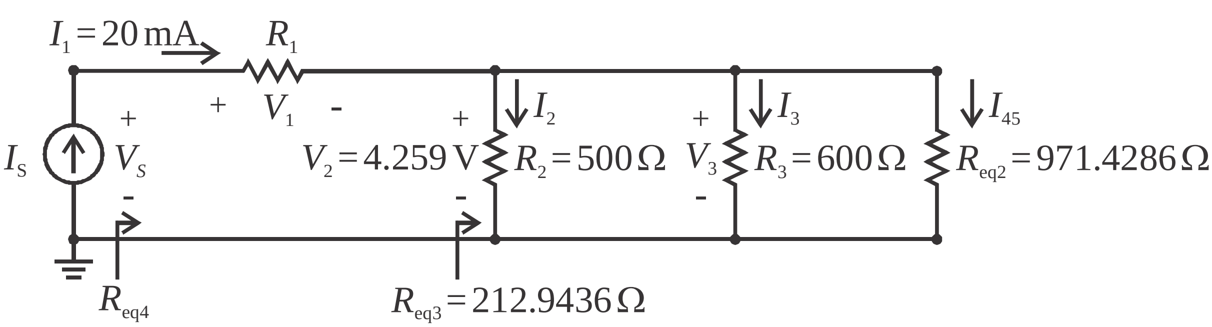
OR

by voltage division, $V_{23} = V_s\,(R_{eq3}/\,R_{eq4}) = 6.25887(212.9/312.9) = 4.25887$ V

So, $\boxed{V_1 = 2\text{ V}}$ and $\boxed{V_{23} = V_2 = V_3 = 4.2589\text{ V}}$.

Step 6

Knowing I_1, V_1, V_2, V_3, & V_S, go back to circuit of step 2 and find I_2, I_3, & I_{45}.



Again, there are multiple ways to calculate I_2, I_3, & I_{45}:

By Ohm's Law, $I_2 = V_2 / R_2 = 4.259 / 500 = 8.51774$ mA,

OR

by current division, $I_2 = I_1 (R_{eq3} / R_2) = 20(212.944 / 500) = 8.51774$ mA.

Next, by Ohm's Law, $I_3 = V_3 / R_3 = 4.259 / 600 = 7.09812$ mA,

OR

by current division, $I_3 = I_1 (R_{eq3} / R_3) = 20(212.944 / 600) = 7.09812$ mA.

Last, by Ohm's Law, $I_{45} = V_3 / R_{eq2} = 4.259 / 971.4286 = 4.38413$ mA,

OR

by current division, $I_{45} = I_1 (R_{eq3} / R_{eq2}) = 20(212.94 / 971.43) = 4.3841$ mA,

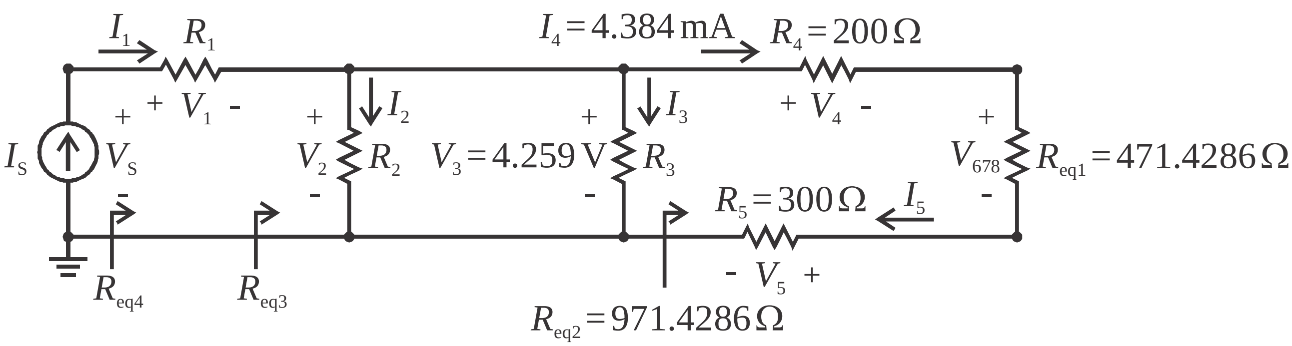
OR

by KCL, $I_1 = I_2 + I_3 + I_{45} \rightarrow I_{45} = 20 - 8.5177 - 7.0981 = 4.3841$ mA.

So, $\boxed{\mathbf{I_2 = 8.5177\ mA}}$, $\boxed{\mathbf{I_3 = 7.0981\ mA}}$, and $\boxed{\mathbf{I_{45} = I_4 = I_5 = 4.3841\ mA}}$.

Step 7

Knowing I_1, I_2, I_3, I_4, I_5, V_1, V_2, V_3, & V_S, go back to circuit of step 1 and find V_4, V_5, & V_{678}.



Again, there are multiple ways to calculate V_4, V_5, & V_{678}:

By Ohm's Law, $V_4 = I_4 R_4 = 4.38414 * 10^{-3}(200) = 0.87683$ V,

OR

by voltage division, $V_4 = V_3 (R_4 / R_{eq2}) = 4.25887(200/971.4286) = 0.87683$ V.

Next, by Ohm's Law, $V_5 = I_5 R_5 = 4.38414 * 10^{-3}(300) = 1.31524$ V,

OR

by voltage division, $V_5 = V_3 (R_5 / R_{eq2}) = 4.25887(300/971.4286) = 1.31524$ V.

Last, by Ohm's Law, $V_{678} = I_4 R_{eq1} = 4.38414*10^{-3}(471.4286) = 2.06681$ V,

OR

by voltage division, $V_{678} = V_3(R_{eq1}/R_{eq2}) = 4.25887(471.4286/971.4286) = 2.066805$ V,

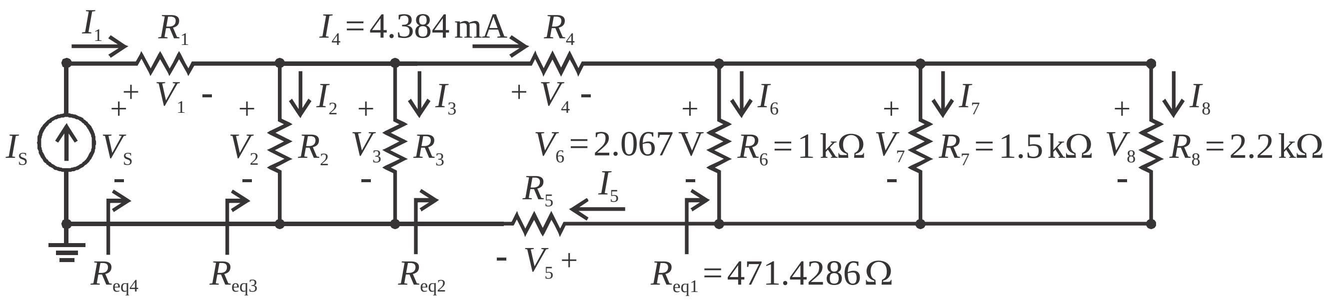
OR

by KVL, $V_3 = V_4 + V_{678} + V_5 \rightarrow V_{678} = 4.25887 - 0.87683 - 1.31524 = 2.0668$ V.

So, $\boxed{V_4 = 0.8768 \text{ V}}$, $\boxed{V_5 = 1.3152 \text{ V}}$, and $\boxed{V_{678} = V_6 = V_7 = V_8 = 2.0668 \text{ V}}$.

Step 8

Knowing I_1, I_2, I_3, I_4, I_5, V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, & V_S, go back to original circuit and find I_6, I_7, & I_8.



Again, there are multiple ways to calculate I_6, I_7, & I_8:

By Ohm's Law, $I_6 = V_6 / R_6 = 2.06681 / 1000 = 2.06681$ mA,

OR

by current division, $I_6 = I_4(R_{eq1}/R_6) = 4.38414(471.4286/1000) = 2.06681$ mA.

Next, by Ohm's Law, $I_7 = V_7 / R_7 = 2.06681 / 1500 = 1.37787$ mA,

OR

by current division, $I_7 = I_4(R_{eq1}/R_7) = 4.38414(471.4286/1500) = 1.37787$ mA.

Last, by Ohm's Law, $I_8 = V_8 / R_8 = 2.06681 / 2200 = 0.93946$ mA,

OR

by current division, $I_8 = I_4(R_{eq1}/ R_8) = 4.38414(471.4286/2200) = 0.93946$ mA,

OR

by KCL, $I_4 = I_6 + I_7 + I_8 \rightarrow I_8 = 4.38414 - 2.06681 - 1.37787 = 0.93946$ mA.

So, $\boxed{I_6 = 2.0668 \text{ mA}}$, $\boxed{I_7 = 1.3779 \text{ mA}}$, and $\boxed{I_8 = 0.9395 \text{ mA}}$.