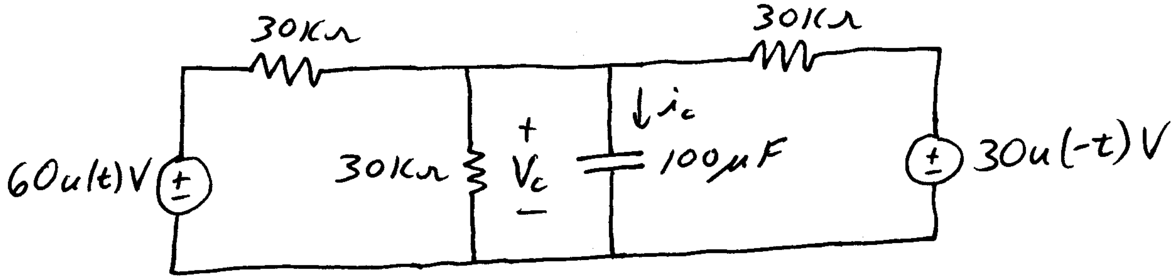
ex. Find $i_c(t)$ and $V_c(t)$ for all $t \geq 0$ for the circuit.



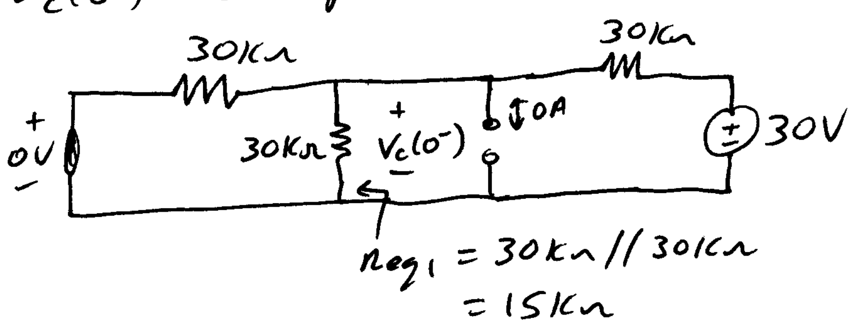
For $t \geq 0$, the circuit undergoes a RC step response as the 60u(t)V turns on and the 30u(−t)V source turns off. Applicable solution form is:

$$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-t/\tau} \quad t \geq 0$$

where $\tau = RC$.

Find $V_c(0)$

→ Analyze circuit at time $t = 0^-$. Why? Circuit is at steady-state. 60u(t)V = 0 (short circuit), 30u(−t)V = 30V (has been on since $t \to -\infty$), and $i_c(0^-) = 0$ (open circuit @ steady-state).



Equiv. Circuit @ $t = 0^-$

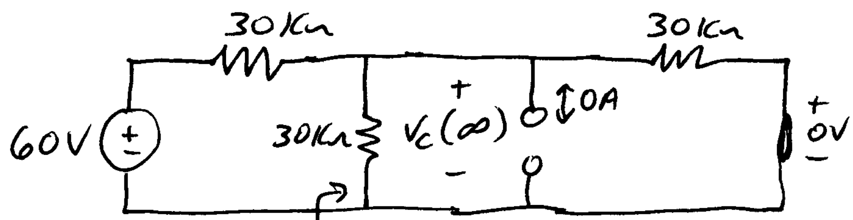
By voltage division $V_{Req1} = V_c(0^-) = 30V \dfrac{15K\Omega}{30K\Omega + 15K\Omega} = \underline{10\,V}$

Capacitor Property $\underline{V_c(0) = V_c(0^-) = 10V}$

ex. cont.

Find $V_c(\infty)$

$\rightarrow$ As $t \rightarrow \infty$, $60u(t)\,V = 60V$ (has been on for all $t>0$), $30u(-t)\,V = 0$ (short circuit), and $i_c(\infty) = 0$ (capacitor is open circuit @ steady-state).



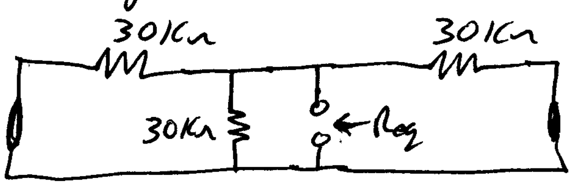
Equiv. circuit as $t \rightarrow \infty$

$R_{eq2} = 30K\Omega \,//\, 30K\Omega = 15K\Omega$

By voltage division $\quad V_{Req2} = V_c(\infty) = 60V \dfrac{15K\Omega}{30K\Omega + 15K\Omega} = \underline{20V}$

Find R, C, & τ for $t>0$

$\rightarrow$ set independent sources to zero & find R_{eq} looking into capacitor terminals



$R_{eq} = 30K\Omega \,//\, 30K\Omega \,//\, 30K\Omega$

$\underline{R_{eq} = 10K\Omega = R}$

$\rightarrow$ $C = 100\mu F$ (No other capacitors)

$\rightarrow$ $\tau = RC = 10 \times 10^3 (100 \times 10^{-6}) = 1\,sec$

Solution

$$V_c(t) = 20 + [10 - 20]e^{-t/1}\ V \quad t \geq 0$$

$$\underline{V_c(t) = 20 - 10e^{-t}\ V \quad t \geq 0}$$

ex. cont.

$$i_c(t) = C\frac{dv_c}{dt}$$

$$= 100\times10^{-6}\ \frac{d(20-10e^{-t})}{dt}$$

$$= 100\times10^{-6}\left[0-10(-1)e^{-t}\right]$$

$$= 0.001\, e^{-t}\ A$$

$$i_c(t) = 1\, e^{-t}\ mA \quad t>0$$

We do NOT know $i_c(t=0)$; slope of $v_c(t=0)$ unknown.

Bonus: $v_c(t<0) = v_c(0^-) = 10\,V$

$$i_c(t<0) = C\frac{dv_c(t<0)}{dt} = 0 \text{ (Steady-state)}$$

$$\boxed{v_c(t) = \begin{cases} 10\,V & t\le 0 \\ 20-10e^{-t}\,V & t\ge 0\end{cases}}$$

$$\boxed{i_c(t) = \begin{cases} 0 & t<0 \\ ? & t=0 \\ 1e^{-t}\,mA & t>0\end{cases}}$$

<u>ex. cont.</u> **Plot $i_C(t)$ and $v_C(t)$**

$n := 0..349$ $\quad t_n := (n-49)\cdot 0.01$

$IC1 := 0$ $\quad VC1 := 10$

$IC2(t) := 1\cdot e^{-t}$ $\quad VC2(t) := 20 - 10\cdot e^{-t}$

$iC_n := \text{if}\left(t_n < 0, IC1, IC2\left(t_n\right)\right)$ $\quad vC_n := \text{if}\left(t_n < 0, VC1, VC2\left(t_n\right)\right)$