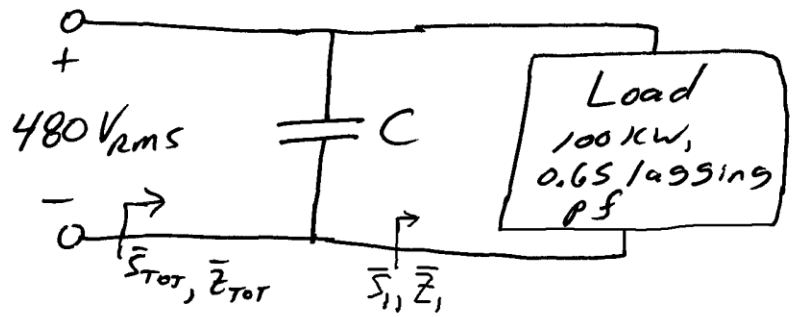
Example- A U.S. industrial plant (f = 60 Hz) is supplied by a 480 V_{RMS} power line and consumes 100 kW (measured using wattmeter) with a power factor of 0.65 lagging. To receive the best electrical rates, a power factor of $pf \geq 0.95$ is required. What range of capacitances connected in parallel will work? What capacitance will yield a $pf = 1$?



Angle associated w/ original $pf = 0.65$ $\quad \theta_1 = \cos^{-1} pf = \cos^{-1}(0.65)$

$= +49.4584^\circ \leftarrow$ lagging

Angle(s) associated w/ desired $pf = 0.95$ $\quad \theta_2 = \cos^{-1} 0.95 = \pm 18.1949^\circ$ ← lagging ← leading

Using eq'n (11.60) $\quad C = \dfrac{P_1(\tan\theta_1 - \tan\theta_2)}{\omega V_{RMS}^2}$

Minimum capacitance (will correct to a $pf = 0.95$ lagging)

$$C_{min} = \frac{100\times10^3(\tan 49.4584^\circ - \tan 18.1949^\circ)}{(2\pi 60)\,480^2}$$

$$\underline{\underline{C_{min} = 967.6\,\mu F = 0.9676\,mF}}$$

Maximum capacitance (will correct to a $pf = 0.95$ leading)

$$C_{max} = \frac{100\times10^3(\tan 49.4584^\circ - \tan(-18.1949^\circ))}{(2\pi 60)\,480^2}$$

$$\underline{\underline{C_{max} = 1724.4\,\mu F = 1.7244\,mF}}$$

$$\boxed{\text{Range: } 0.9676\,mF \le C \le 1.7244\,mF}$$

<u>Capacitance for a pf = 1</u>

<u>Method 1</u> $\theta_2 = \cos^{-1} 1 = 0°$

use (11.60)

$$C_{pf=1} = \frac{100\times10^3(\tan 49.4584° - \tan 0)}{(2\pi 60)\,480^2}$$

$$\underline{\underline{C_{pf=1} = 1346\,\mu F = 1.346\,mF}}$$

$\hookrightarrow$ $\underline{\bar{S}_{TOT} = P_1 = 100\,KW}$ (no reactive power!)

<u>Method 2</u> $\rightarrow$ Make $\bar{Z}_{TOT}$ &/or $\bar{Y}_{TOT}$ entirely <u>real</u>

$(\theta_2 = 0° \Rightarrow pf = \cos\theta_2 = 1)$

$$P_{ave} = V_{rms} I_{rms} \cos\theta_1 \Rightarrow I_{rms} = \frac{100\times10^3}{480(0.65)} = 320.5128\ A_{rms}$$

$$|\bar{Z}_1| = \frac{V_{rms}}{I_{rms}} = \frac{480}{320.5} = 1.4976\ \Omega$$

$$\measuredangle \bar{Z}_1 = \cos^{-1} 0.65 = 49.4584°$$

$$\bar{Z}_1 = 1.4976\angle 49.4584°\ \Omega \longrightarrow \bar{Y}_1 = \frac{1}{\bar{Z}_1} = 0.434 - j0.5074\ S$$

Add parallel admittances $\Rightarrow \bar{Y}_{TOT} = \bar{Y}_1 + \bar{Y}_c$ = Real #

To make this occur $-j0.5074 + j\omega C = 0$

$\hookrightarrow$ $C_{pf=1} = \dfrac{0.5074}{2\pi(60)} = \underline{\underline{1.346\,mF}}$ $\leftarrow$ Same!