Example- An unknown logic circuit with 4 inputs has the Truth Table (below) when tested. Earlier, we used a four variable K-Map to determine Boolean functions in the canonical sum-of-minterms and simplified sum-of-products forms for this unknown logic circuit.

a	*b*	*c*	*d*	F_2
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**1**

- The K-Map (below) was used to directly express F_2 in canonical sum-of-minterms form (**eight** terms and **32** literals).

ab\cd	**00**	**01**	**11**	**10**
00	~~m_0~~	~~m_1~~	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	~~m_4~~	~~m_5~~	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	~~m_{14}~~
10	~~m_8~~	~~m_9~~	~~m_{11}~~	$m_{10}=a\,b'c\,d'$

$$F_2 = \sum(2,3,6,7,10,12,13,15)$$
$$= a'b'c\,d' + a'b'c\,d + a'b\,c\,d' + a'b\,c\,d + a\,b'c\,d' + a\,b\,c'd' + a\,b\,c'd + a\,b\,c\,d$$

- Then, we used this K-Map to directly express F_2 in a simplified sum-of-products form which gave two answers.

$$F_{2,\text{simp1}} = a'c + b'cd' + abc' + bcd$$

or

$$F_{2,\text{simp2}} = a'c + b'cd' + abc' + abd$$

Now, we wish to express F_2 in canonical product-of-maxterms and simplified product-of-sums forms using K-Maps.

- To begin, we fill-in the K-Map from the Truth Table with "1"s and "0"s.

ab\cd	**00**	**01**	**11**	**10**
00	0	0	1	1
01	0	0	1	1
11	1	1	1	0
10	0	0	0	1

- Next, we fill-in the minterms where there are "0"s and use the K-Map to directly express F_2' in canonical sum-of-minterms form (**eight** terms and **32** literals).

ab\cd	**00**	**01**	**11**	**10**
00	$m_0 = a'b'c'd'$	$m_1 = a'b'c'd$	~~m_3~~	~~m_2~~
01	$m_4 = a'bc'd'$	$m_5 = a'bc'd$	~~m_7~~	~~m_6~~
11	~~m_{12}~~	~~m_{13}~~	~~m_{15}~~	$m_{14} = abcd'$
10	$m_8 = ab'c'd'$	$m_9 = ab'c'd$	$m_{11} = ab'cd$	~~m_{10}~~

$$F_2' = \sum(0,1,4,5,8,9,11,14)$$
$$= a'b'c'd' + a'b'c'd + a'bc'd' + a'bc'd + ab'c'd' + ab'c'd + ab'cd + abcd'$$

- Then, use this K-Map to directly express F_2' in a simplified sum-of-products form.

ab\cd	**00**	**01**	**11**	**10**
00	$m_0=a'b'c'd'$	$m_1=a'b'c'd$	~~m_3~~	~~m_2~~
01	$m_4=a'bc'd'$	$m_5=a'bc'd$	~~m_7~~	~~m_6~~
11	~~m_{12}~~	~~m_{13}~~	~~m_{15}~~	$m_{14}=abcd'$
10	$m_8=ab'c'd'$	$m_9=ab'c'd$	$m_{11}=ab'cd$	~~m_{10}~~

a) The upper left corner has <u>four</u> adjacent squares (m_0, m_1, m_4, & m_5) which share the pair of literals/prime implicant $a'c'$ [we eliminated literals b, b', d, & d']. Note that the prime implicant $a'c'$ is <u>essential</u> for minterms m_4 and m_5.

ab\cd	**00**	**01**	**11**	**10**
00	$m_0=a'b'c'd'$	$m_1=a'b'c'd$	~~m_3~~	~~m_2~~
01	$m_4=a'bc'd'$	$m_5=a'bc'd$	~~m_7~~	~~m_6~~
11	~~m_{12}~~	~~m_{13}~~	~~m_{15}~~	$m_{14}=abcd'$
10	$m_8=ab'c'd'$	$m_9=ab'c'd$	$m_{11}=ab'cd$	~~m_{10}~~

b) The upper & lower left corners have <u>four</u> adjacent squares (m_0, m_1, m_8, & m_9) which share the pair of literals/prime implicant $b'c'$ [we eliminated literals a, a', d, & d']. Note that the prime implicant $b'c'$ is <u>essential</u> for minterm m_8 and that minterms m_0 & m_1 are used again.

ab\cd	**00**	**01**	**11**	**10**
00	$m_0=a'b'c'd'$	$m_1=a'b'c'd$	~~m_3~~	~~m_2~~
01	$m_4=a'bc'd'$	$m_5=a'bc'd$	~~m_7~~	~~m_6~~
11	~~m_{12}~~	~~m_{13}~~	~~m_{15}~~	$m_{14}=abcd'$
10	$m_8=ab'c'd'$	$m_9=ab'c'd$	$m_{11}=ab'cd$	~~m_{10}~~

c) The row where $ab = 10$ has three minterms/squares in a row (m_8, m_9, & m_{11}). Since three is not a power of two and we have covered m_8 & m_9 already, we will group the two adjacent minterms/ squares m_9 & m_{11} which share the trio of literals/prime implicant $a\,b'd$ [eliminated literals c & c']. Note that the prime implicant $a\,b'd$ is essential for minterm m_{11}, but not for m_9.

ab\cd	00	01	11	10
00	$m_0=a'b'c'd'$	$m_1=a'b'c'd$	~~m_3~~	~~m_2~~
01	$m_4=a'b\,c'd'$	$m_5=a'b\,c'd$	~~m_7~~	~~m_6~~
11	~~m_{12}~~	~~m_{13}~~	~~m_{15}~~	$m_{14}=a\,b\,c\,d'$
10	$m_8=a\,b'c'd'$	$m_9=a\,b'c'd$	$m_{11}=a\,b'c\,d$	~~m_{10}~~

d) The last remaining minterm/square $m_{14}=a\,b\,c\,d'$ is not adjacent to any other minterm/square. Therefore, it cannot be simplified.

e) Therefore, the simplified sum-of-products forms for F_2' is-

$$F_{2,\text{simp}}' = a'c' + b'c' + a\,b'd + a\,b\,c\,d'$$

This simplified Boolean function has **four** terms and **11** literals [as opposed to **eight** terms and **32** literals].

- Now, we can take the complement of $F_{2,\text{simp}}'$ to express F_2 in a simplified product-of-sums form by using DeMorgan's Theorem (i.e., take dual and then complement each literal).

$$F_{2,\text{simp}} = \left(F_{2,\text{simp}}'\right)' = \left(a'c' + b'c' + a\,b'd + a\,b\,c\,d'\right)'$$
$$= (a+c)(b+c)(a'+b+d')(a'+b'+c'+d)$$

Note, like the simplified sum-of-products forms of F_2, the simplified product-of-sums form also **four** terms and **11** literals [as opposed to **eight** terms and **32** literals].

Check this result using a Truth Table to show that it agrees.

$$F_{2,\text{simp}} = (a+c)(b+c)(a'+b+d')(a'+b'+c'+d)$$

a	b	c	d	a'	b'	c'	d'	$a+c$	$b+c$	$a'+b+d'$	$a'+b'+c'+d$	$F_{2,\text{simp}}$	F_2
0	0	0	0	1	1	1	1	0	0	1	1	**0**	**0**
0	0	0	1	1	1	1	0	0	0	1	1	**0**	**0**
0	0	1	0	1	1	0	1	1	1	1	1	**1**	**1**
0	0	1	1	1	1	0	0	1	1	1	1	**1**	**1**
0	1	0	0	1	0	1	1	0	1	1	1	**0**	**0**
0	1	0	1	1	0	1	0	0	1	1	1	**0**	**0**
0	1	1	0	1	0	0	1	1	1	1	1	**1**	**1**
0	1	1	1	1	0	0	0	1	1	1	1	**1**	**1**
1	0	0	0	0	1	1	1	1	0	1	1	**0**	**0**
1	0	0	1	0	1	1	0	1	0	0	1	**0**	**0**
1	0	1	0	0	1	0	1	1	1	1	1	**1**	**1**
1	0	1	1	0	1	0	0	1	1	0	1	**0**	**0**
1	1	0	0	0	0	1	1	1	1	1	1	**1**	**1**
1	1	0	1	0	0	1	0	1	1	1	1	**1**	**1**
1	1	1	0	0	0	0	1	1	1	1	0	**0**	**0**
1	1	1	1	0	0	0	0	1	1	1	1	**1**	**1**