Example- An unknown logic circuit with 4 inputs results in the Truth Table (below) when tested. We will use a four variable K-Map to determine Boolean functions in the canonical sum-of-minterms and simplified sum-of-products forms for this unknown logic circuit.

a	*b*	*c*	*d*	*F*
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**1**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**1**

- First create K-Map directly from Truth Table.

  a) The first two instances where *F* = 1 correspond to the intersection of the second row where *ab* = 00 and the columns where *cd* = 10 & *cd* = 11.

ab\cd	**00**	**01**	**11**	**10**
00			**1**	**1**
01				
11				
10				

b) The next two instances where F = 1 corresponds to the intersection of the second row where ab = 01 and the columns where cd = 10 & cd = 11.

ab\cd	**00**	**01**	**11**	**10**
00			**1**	**1**
01			**1**	**1**
11				
10				

c) The next instance where F = 1 corresponds to the intersection of the bottom row where ab = 10 and the column where cd = 10.

ab\cd	**00**	**01**	**11**	**10**
00			**1**	**1**
01			**1**	**1**
11				
10				**1**

d) The last three instances where F = 1 corresponds to the intersection of the third row where ab = 11 and the columns where cd = 00, 01 & 11.

ab\cd	**00**	**01**	**11**	**10**
00			**1**	**1**
01			**1**	**1**
11	**1**	**1**	**1**	
10				**1**

- Next, use this K-Map to directly express F in the canonical sum-of-minterms form. Put each applicable minterm into each square where there was a “1”-

ab\cd	00	01	11	10
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

Therefore,

$$F=\sum(2,3,6,7,10,12,13,15)$$
$$=m_2+m_3+m_6+m_7+m_{10}+m_{12}+m_{13}+m_{15}$$
$$=a'b'c\,d'+a'b'c\,d+a'b\,c\,d'+a'b\,c\,d+a\,b'c\,d'+a\,b\,c'd'+a\,b\,c'd+a\,b\,c\,d$$

which has **eight** terms and **32** literals.

- Then, use this K-Map to directly express F in a simplified sum-of-products form.

ab\cd	00	01	11	10
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

a) The upper right corner has <u>four</u> adjacent minterms/squares (m_2, m_3, m_6, & m_7) which share the duo of literals/prime implicant $a'c$ [we eliminated literals b, b', d, & d']. Note that the prime implicant $a'c$ is <u>essential</u> for minterms m_3 & m_6.

ab\cd	00	01	11	10
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

b) The upper & lower right corners have <u>two</u> adjacent squares (m_2 & m_{10}) which share the trio of literals/prime implicant $b'c\,d'$ [we eliminated literals a & a']. Note that the prime implicant $b'c\,d'$ is <u>essential</u> for minterm m_{10}, but not for m_2.

ab\cd	00	01	11	10
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

c) The ab = 11 row has <u>three</u> squares in a row (m_{12}, m_{13} & m_{15}). Since three is not a power of two, we group the <u>two</u> adjacent minterms m_{12} & m_{13} which share the trio of literals/prime implicant $a\,b\,c'$ [eliminated literals d & d']. Note that the prime implicant $a\,b\,c'$ is <u>essential</u> for minterm m_{12}, but not for m_{13}.

d) The remaining minterm m_{15} can be dealt with in a couple ways:

ab\cd	00	01	11	10
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

1) pair-up m_7 & m_{15} which share the trio of literals/prime implicant $b\,c\,d$ [We eliminated literals a & a'. Minterm m_7 is used twice. Note that the prime implicant $b\,c\,d$ is <u>essential</u> for minterm m_{15}, but not for m_7.], or

ab\cd	**00**	**01**	**11**	**10**
00	$\cancel{m_0}$	$\cancel{m_1}$	$m_3=a'b'c\,d$	$m_2=a'b'c\,d'$
01	$\cancel{m_4}$	$\cancel{m_5}$	$m_7=a'b\,c\,d$	$m_6=a'b\,c\,d'$
11	$m_{12}=a\,b\,c'd'$	$m_{13}=a\,b\,c'd$	$m_{15}=a\,b\,c\,d$	$\cancel{m_{14}}$
10	$\cancel{m_8}$	$\cancel{m_9}$	$\cancel{m_{11}}$	$m_{10}=a\,b'c\,d'$

2) pair-up m_{13} & m_{15} which share the trio of literals/prime implicant $a\,b\,d$. We eliminated literals c & c'. Minterm m_{13} is used twice. Note that the prime implicant $a\,b\,d$ is <u>essential</u> for minterm m_{15}, but not for m_{13}.

Options 1 & 2 are equally good choices from the standpoint of simplification and minimizing gates

e) Therefore, the simplified sum-of-products forms for F are-

$$F_{\text{simp1}} = a'c + b'c\,d' + a\,b\,c' + b\,c\,d$$

OR

$$F_{\text{simp2}} = a'c + b'c\,d' + a\,b\,c' + a\,b\,d$$

These simplified Boolean functions have **four** terms and **11** literals [as opposed to **eight** terms and **32** literals].

- Check these results using Truth Tables to show they agree.

$$F_{\text{simp1}} = a'c + b'cd' + abc' + bcd$$

a	b	c	d	a'	b'	c'	d'	$a'c$	$b'cd'$	abc'	bcd	F_{simp1}	F
0	0	0	0	1	1	1	1	0	0	0	0	**0**	**0**
0	0	0	1	1	1	1	0	0	0	0	0	**0**	**0**
0	0	1	0	1	1	0	1	1	1	0	0	**1**	**1**
0	0	1	1	1	1	0	0	1	0	0	0	**1**	**1**
0	1	0	0	1	0	1	1	0	0	0	0	**0**	**0**
0	1	0	1	1	0	1	0	0	0	0	0	**0**	**0**
0	1	1	0	1	0	0	1	1	0	0	0	**1**	**1**
0	1	1	1	1	0	0	0	1	0	0	1	**1**	**1**
1	0	0	0	0	1	1	1	0	0	0	0	**0**	**0**
1	0	0	1	0	1	1	0	0	0	0	0	**0**	**0**
1	0	1	0	0	1	0	1	0	1	0	0	**1**	**1**
1	0	1	1	0	1	0	0	0	0	0	0	**0**	**0**
1	1	0	0	0	0	1	1	0	0	1	0	**1**	**1**
1	1	0	1	0	0	1	0	0	0	1	0	**1**	**1**
1	1	1	0	0	0	0	1	0	0	0	0	**0**	**0**
1	1	1	1	0	0	0	0	0	0	0	1	**1**	**1**

$$F_{\text{simp2}} = a'c + b'cd' + abc' + abd$$

a	b	c	d	a'	b'	c'	d'	$a'c$	$b'cd'$	abc'	abd	F_{simp2}	F_2
0	0	0	0	1	1	1	1	0	0	0	0	**0**	**0**
0	0	0	1	1	1	1	0	0	0	0	0	**0**	**0**
0	0	1	0	1	1	0	1	1	1	0	0	**1**	**1**
0	0	1	1	1	1	0	0	1	0	0	0	**1**	**1**
0	1	0	0	1	0	1	1	0	0	0	0	**0**	**0**
0	1	0	1	1	0	1	0	0	0	0	0	**0**	**0**
0	1	1	0	1	0	0	1	1	0	0	0	**1**	**1**
0	1	1	1	1	0	0	0	1	0	0	0	**1**	**1**
1	0	0	0	0	1	1	1	0	0	0	0	**0**	**0**
1	0	0	1	0	1	1	0	0	0	0	0	**0**	**0**
1	0	1	0	0	1	0	1	0	1	0	0	**1**	**1**
1	0	1	1	0	1	0	0	0	0	0	0	**0**	**0**
1	1	0	0	0	0	1	1	0	0	1	0	**1**	**1**
1	1	0	1	0	0	1	0	0	0	1	1	**1**	**1**
1	1	1	0	0	0	0	1	0	0	0	0	**0**	**0**
1	1	1	1	0	0	0	0	0	0	0	1	**1**	**1**